Example Problem Involving both Momentum and Energy

In moving from X to Y on a level surface, Block A encounters no friction as it moves toward stationary Block B at constant velocity, \( v_0 \). The two blocks of equal mass M stick together in the collision. During the small time \( t \) that the collision lasts, the blocks move a negligible distance. After the collision, the blocks slide together through and beyond region YZ. In region YZ, which is a distance \( d \) in length, the track exerts a constant friction force \( f \) to the left on the combined blocks. Express answers in simplest form in terms of the given symbols \( M, v_0, d, f, \) and \( t \).

a. Determine the velocity \( v \) of the combined blocks immediately after the collision.

**System:** blocks A, B, and the forces they exert on each other

**External forces:** Normal and gravity add to 0 so \( F_{net,ext} = 0 \) and momentum is conserved. We’re assuming that in the short time that the collision lasts, friction doesn’t have a significant effect.

**States:** initial – just before collision  final – just after collision

Since \( F_{net,ext} = 0 \), we can start with \( P_i = P_f \).

\[
P_i = P_f,
\]

\[
P_{Ai} + P_{Bi} = P_{Af} + P_{Bf}
\]

\[
Mv_0 + 0 = 2Mv
\]

\[
v = v_0 / 2
\]

b. What percentage of block A’s kinetic energy before the collision do the combined blocks have immediately after the collision?

We’re looking for the quantity \( 100K_f/K_i \).

\[
%K = 100\frac{K_f}{K_i}
\]

\[
= 100\left(\frac{1}{2}Mv^2\right) / \left(\frac{1}{2}2Mv_0^2\right)
\]

\[
= 100\frac{2(v_0/2)^2}{v_0^2}
\]

\[
= 100/4
\]

\[
= 25
\]

c. Determine the magnitude of the average force that each block exerts on the other during the collision.

We’ll use the impulse-momentum relationship: \( F\Delta t = \Delta p \). We can solve in either of two ways. For the force that A exerts on B, we have the following. Note that we calculate the momentum of change of block B in order to get the force on block B.

\[
F_{AonB} = \Delta p_B / \Delta t
\]

\[
= M \Delta v_B / t
\]

\[
= M (v_f / 2 - 0) / t
\]

\[
= Mv_f / (2t)
\]

Alternatively, we can calculate the momentum change of block A in order to get the force on block A. We know by Newton’s 3rd Law that the force of B on A will have the same magnitude as the force of A on B.

\[
F_{Bonda} = \Delta p_A / \Delta t
\]

\[
= M \Delta v_A / t
\]

\[
= M (v_i / 2 - v_f) / t
\]

\[
= -Mv_f / (2t)
\]

Since we’re only interested in magnitudes, we get the same result as previously.
d. Determine the speed $v_z$ of the combined blocks at Point Z.

This can be done using strictly net force-dvat methods, but we’ll use energy methods, since that’s our topic.

**System:** blocks A, B and forces they exert on each other

**External forces:** Normal and gravity do no work since the forces are perpendicular to the displacement of the blocks. However, kinetic friction does work on the blocks.

**States:** initial – blocks just after the collision        final - blocks reaching point Z

**Energy changes:** $K$ decreases. There is no potential energy.

The work done by friction is $W_f = f d \cos \theta$, where $\theta = 180^\circ$. Therefore, $W_f = -fd$.

$$W_{ext} = \Delta K$$
$$W_f = \frac{1}{2}(2M)(v_f^2 - v_i^2)$$
$$-fd = M(v_z^2 - (v_o / 2)^2)$$
$$Mv_z^2 = Mv_o^2 / 4 - fd$$
$$v_z^2 = v_o^2 / 4 - fd / M$$
$$v_z = (v_o^2 / 4 - fd / M)^{0.5}$$

A check on this result is to examine the case of no friction, $f = 0$. In that case, $v_z = v_o/2$. This makes sense, because the blocks wouldn’t slow down in the absence of friction.