Study Tips about reading. The article that follows isn’t necessarily easy reading, but it should be interesting. It’s designed to make you think. Since this is your first reading assignment on scientific content, this is a good time for some advice on how to read scientific material. If you form good habits now, they'll work to your benefit throughout the course. Here are some guidelines.

1. Have a writing instrument and notebook available.
2. Remember that you're reading for understanding. That means you need to do your best to understand the content that's being presented. If you don't understand something, make a note in your notebook to ask about it in the Question & Answer forum. Also, write down the important ideas in the reading. You can write in the form of an outline or a list, but the important thing is to take notes in a way that you can easily refer to later.
3. Don't gloss over the equations and diagrams. Go through the equations step-by-step. Examine the diagrams in order to determine what information they convey.
4. Study the example problems. These are your guides to solving problems. They're more important than for a regular classroom, because in an online course, the teacher won't be doing example problems at the board.

Why Ants are Stronger than Humans, Relatively Speaking

You may have heard it said that an ant can carry a weight several times its own weight. You’ve probably observed this weight-lifting feat at a picnic when ants carry crumbs of food away. Why can ants do—as part of their daily routine—something that humans can’t? Answering this question is a problem of scaling.

Scaling of Area

We begin by looking at some familiar geometric relationships. Consider the relationship between length and area. Line $L_2$ below is twice as long as Line $L_1$. That is, $L_2 = 2L_1$.

![Diagram of two squares with sides $L_1$ and $L_2$ and areas $A_1$ and $A_2$.]

Suppose that these lines are used as the sides of two squares. The areas of these two squares are $A_1 = L_1^2$ and $A_2 = L_2^2$. We can form the following ratio between the areas:

$$
\frac{A_2}{A_1} = \frac{L_2^2}{L_1^2} = \frac{(2L_1)^2}{L_1^2} = \frac{4L_1^2}{L_1^2} = 4.
$$

While the lengths of the sides are in the ratio of 2:1, the areas are in the ratio of 4:1. This makes sense, because we could fit 4 of the smaller square inside the larger as shown to the right.
In general, if \( L_2/L_1 = k \), then one can replace 2 with \( k \) in the equation above and show that:

\[ \frac{A_2}{A_1} = k^2. \]

This is called a scaling relationship. In words, we say that the area scales as the square of the length of the side. We could also call \( k \) a scale factor and say that area increases as the square of the scale factor. We will use the term scale factor to represent a ratio of linear dimensions of 2 similar objects.

Saying that the area of a square increases as the square of the scale factor is just a different way of saying something that you already knew about squares. The interesting thing, however, is that we can apply it to all areas for which we can identify a characteristic length. For example, the characteristic length of a circle is its radius (or diameter). A circle with twice the radius of another has four times the area. The reason this scaling relationship works the same for a circle as for a cube is that the factor of \( \pi \) that is used to calculate area divides out as shown below. We use \( R_1 \) and \( R_2 \) to represent the two radii.

\[
\frac{A_2}{A_1} = \frac{R_2^2}{R_1^2} = \frac{(2R_1)^2}{R_1^2} = \frac{4R_1^2}{R_1^2} = 4
\]

In general, a circle with \( k \) times the radius of another has \( k^2 \) times the area. This conclusion will be used in the next section. Note that we can substitute the word diameter for radius in the above discussion, because the diameter and radius differ only by a constant factor of 2. Constant factors divide out when forming ratios such as those above.

**Example Problem 1.** The diameter of a cylinder is doubled. By what factor does the volume change?

**Solution.** The volume of a cylinder is the area of its cross section times its height. The cross-sectional area is proportional to the square of the diameter. Doubling the diameter will therefore quadruple the area. The height is unchanged, so the volume is also quadrupled.

**Scaling of muscle and bone strength**

Let’s look at what factors determine the strength of muscle tissue. Consider a fiber such as a thread. If you pull on the thread hard enough, it will eventually break. Does the amount of force required to break it depend on how long the thread is? We claim that it doesn’t, but you can try the test yourself if you’re not sure. Tie one end of a length of thread to a support and then start hanging weights to the other end until the thread breaks. (Be sure to put a cushion on the floor to catch the weights when they fall!) Try again with both longer and shorter thread. All of them should break with about the same amount of weight. Don’t expect the weights to be identical, though, because all threads aren’t identical, even when taken from the same spool.

One thing that does influence the amount of force required to break the thread is the number of threads supporting the weight. The greater the number of threads, the greater the weight can be. In fact, the breaking force and number of threads are proportional. That is, two threads require twice the breaking force, or \( N \) threads require \( N \times \) the breaking force.

\[ \text{Breaking force} \propto \text{Number of threads, } N \]

(In case you’re not familiar with the symbol, \( \propto \), it means is proportional to.)
You could test this relationship by experiment. Put two lengths of thread side-by-side and suspend weights from the pair of strings. As in the previous experiment, see how much weight the pair can hold. Then repeat with, say, three threads side-by-side. If one wanted to do a systematic experiment, one could try 4 and 5 threads also and then plot a graph of the breaking force versus the number of threads. The result might look like the graph to the right.

The characteristic of the thread or bundle of threads that changes whenever more threads are added is the total cross-sectional area of the bundle. A bundle of N threads has N times the cross-sectional area of a single thread and can hold N times as much weight. Therefore, we can say

\[
\text{Breaking force } \propto N \cdot (\text{Cross-sectional area of 1 thread})
\]

\[
\propto \text{Total cross-sectional area of N threads}
\]

What does this have to do with ants and humans? Simply that ant muscles and human muscles are similar in that they’re composed of bundles of fibers. The more fibers there are in a bundle, the stronger the muscle will be. Of course, the kind of tissue that makes up the fiber is important, too, in the same way that the material that a thread is composed of affects its strength. A metal fiber of the same diameter as a cotton thread may be stronger, because the molecules of the material bond more tightly together. However, such differences in composition are not important in comparing ant and human muscles, because the muscle tissue has a similar composition for both.

Let’s combine the previous result for the scaling of breaking force with that obtained for the scaling of area. Since area increases as the square of the scale factor and breaking force increases as the total cross-sectional area, then breaking force increases as the square of the scale factor.

\[
\text{Breaking force } \propto k^2
\]

Similar considerations apply for the strength of skeletal structures such as bones for humans and exoskeletons for ants. The supporting strength of these structures depends on their cross-sectional area, which depends, in turn, on the square of the scale factor. This is true whether the force of interest is tensile, as in pulling on strings or muscles, or compressive, as in pushing on bones.

**Example Problem 2.** Wires A and B are made of the same material, but wire B has 8 times the diameter of wire A. How many times more weight can wire B support without breaking than wire A?

**Solution.** The strength of a wire is proportional to its area of cross section, which in turn is proportional to the square of the diameter. Wire B has \(8^2\) times the cross-sectional area as wire A. Therefore, wire B can support 64 times as much weight.
From now on, we’ll use the term strength instead of breaking force, since strength is a familiar way of talking about muscles. Therefore, the strength of a bundle of muscle fibers or of a bone is proportional to the square of the scale factor. One might be thinking at this point that humans are obviously stronger than ants, because humans have legs of larger diameter (and cross-sectional area) than ants. Of course that’s correct and something we already knew. However, the question we’re interested in is how much humans and ants can carry in proportion to their own weight. So next we need to look at what factors affect weight (other than eating at McDonald’s).

Scaling of Volume

We looked at relationships for area earlier and found that area scales as the square of the scale factor. Next, let’s look at volume. Let’s compare the volumes, \( V_1 \) and \( V_2 \), of two cubes with sides \( L_1 \) and \( L_2 \).

\[
\frac{V_2}{V_1} = \frac{L_2^3}{L_1^3} = \frac{(2L_1)^3}{L_1^3} = \frac{8L_1^3}{L_1^3} = 8
\]

When the length of the side is doubled, the volume is 8 times greater. In general, a cube with \( k \) times the side of another has \( k^3 \) times the volume. Similar reasoning applies to the volume of a sphere. Consider two spheres of radii \( R_1 \) and \( R_2 \) where \( R_1 = 2 \times R_2 \).

\[
\frac{V_2}{V_1} = \frac{(4\pi/3)R_2^3}{(4\pi/3)R_1^3} = \frac{(2R_1)^3}{R_1^3} = \frac{8R_1^3}{R_1^3} = 8
\]

The only difference between this situation and that of a cube is the factor of \( 4\pi/3 \), which divides out. In general, we say that volume increases as the cube of the scale factor. This means that in comparing two volumes of a particular shape, we don’t have to use the formula for volume for that shape. We just have to know the ratio of some characteristic linear dimension and then cube the ratio. For example, suppose that we have two balloons of the same kind and blow them up to different sizes. Let’s say the distance across at the widest part of the bulge is 3 times greater for one balloon than the other. For any characteristic distance that we pick, such as the distance from knot to bottommost point, the ratio would also be 3 or very nearly so. (We’re assuming that the balloon expands in the same way in every direction.) Thus, we can immediately say that the ratio of the volumes of the two balloons is \( (3)^3 = 27 \). The power of this scaling relationship is that we can compare volumes without having a formula to calculate the volume of a balloon. We say that volume increases as the cube of the scale factor. In the balloon example, the scale factor is 3.

Such scaling relationships are great time savers and are commonly used in engineering for designing structures and scaling up from models to a full-size structure. Galileo first studied scaling systematically in the 17th century in comparing the strengths of supporting beams. Scaling is a tool used in many areas of science, including such diverse fields as aerodynamics and biophysics. So it’s not as unusual as one might think to compare ants to humans.

Example Problem 3. Sphere A has a diameter of 6 cm and a volume of 113 cm\(^3\). Sphere B has a diameter of 18 cm. How many times bigger is the volume of sphere B than the volume of sphere A? What is the volume of sphere B?
Solution. Volume is proportional to the cube of the diameter. Note that B's diameter is 3 times that of A's. Therefore, B's volume is \(3^3 = 27\) times that of A's. Therefore, the volume of B is \(27 \times 113 \text{ cm}^3 = 3051 \text{ cm}^3\). Note that we didn’t have to calculate the volume of sphere A. That’s an advantage of using scale factors.

Example Problem 4. 216 small cubical blocks will just fill up a large cubical box. If the length of a side of a small block is 2 inches, what is the length of a side of the large box?

Solution. We know that the large box has 216 times the volume of a block, since blocks can be packed to totally fill the space inside the box. Since volume is proportional to the cube of a side, the side of a small block is \(216^{1/3} = 6\) times that of the box. Therefore, the side of the box is \(6 \times 2\) inches = 12 inches.

Scaling of Weight

Now that we have a scaling relationship for volume, let’s extend it to one for weights. The weight of a certain volume of a substance depends on the volume. For example, suppose we have a spherical lump of clay of a particular weight and volume. If we deform the clay into a different shape, it still has the same volume. You can test this by dropping the clay in a glass of water both before and after the clay is deformed. The water level will rise by the same amount each time. Of course, the clay will also have the same weight before and after being deformed. If we now take a second spherical lump of clay identical in size to the first lump, the two lumps together will have twice the weight and twice the volume as a single lump. If squashed together to form one lump, the total weight will simply be the sum of the weights of the two lumps. Likewise, the total volume will be the sum of the volumes of the two lumps. We conclude that weight and volume are proportional to each other.

\[
\text{Weight} \; \propto \; \text{Volume}
\]

An important consideration in this argument is that the composition of the objects whose weights and volumes we’re comparing is the same. For example, equal volumes of clay and marshmallow would obviously have different weights. (You may already know that the relevant property that distinguishes two substances such as clay and marshmallow in the above argument is the density of the substance.)

In the last section, we found that volume increases as the cube of the scale factor. Since weight is proportional to volume, then weight also increases as the cube of the scale factor.

\[
\text{Weight} \; \propto \; k^3
\]

Consider the same two balloons as discussed in the previous section, but let’s fill them with water instead of air. We found that the larger balloon had \(3^3 = 27\) times the volume of the smaller. Therefore, the water in the larger balloon also weighs \(27\) times as much as the smaller.

Example Problem 5. A cannonball weighs 200 times more than a musket ball. If both are made of lead, how do their diameters compare?

Solution. Since weight is proportional to volume for objects of the same density, the cannonball must have 200 times the volume of the musket ball. Since volume is
proportional to the cube of the diameter and the ratio of volumes is 200/1, the ratio of diameters is \( \frac{200}{1} \)^{1/3} = 5.85.

Now we’re ready to compare the weights of humans and ants. We’re first going to assume that the material composing humans and ants is of about the same composition overall. (We know that there’s water and organic material in both.) That’s another of our approximations, and it allows us to say that weight increases as the cube of the scale factor. We need yet another approximation. Ants and humans have different shapes, but we need to imagine the same shape for both. One way to approach this is to approximate the human shape as, say, a cylinder. This applies fairly well to the human torso, which is where much of the weight of the body is. We can apply a similar approximation to the shape of an ant by imagining the head, thorax, and abdomen as blending into a cylinder. This gives us a way to obtain a scale factor by comparing the height of our cylindrical human to the length of a cylindrical ant.

You may be bothered by all the approximations we’ve been making. You needn’t be. Knowing when and how to approximate is an important scientific skill. This makes it possible to do problems that otherwise seem too complicated. Of course, we can’t expect to get exact results, but then we never expect to get exact results in science. We do the best we can with the tools that we have.

Returning to ants and humans, we need a scale factor to relate their sizes so that we can then relate their weights. Let’s take the height of our cylindrical human to be 100 cm and the length of our cylindrical ant to be 1 cm. Then the scale factor is \( k = \frac{100}{1} = 100 \). This means that the volume and weight of the human are about \( 100^3 = 10^6 \) times greater than that of the ant.

Now we’re ready to look at muscle and bone strength in relation to weight.

**Answering the Original Question**

Let’s first summarize what we’ve found about scaling relationships.

1. The strength of muscles and skeletal structures (that is, their ability to support weight) increases as the cross-sectional area of the muscle or skeletal structure.
2. Area increases as the square of the scale factor.
3. Volume increases as the cube of the scale factor.
4. The weight of objects of the same material increases as the cube of the scale factor.

In order to see how much more weight an ant could support than a human in relation to its size, let’s take an ordinary ant and scale it up to human size. We’ll use the scale factor of \( k = 100 \) that we found earlier. Since strength increases as the square of the scale factor, the human-sized ant can support \( 10^4 = 10,000 \) times as much weight. However, the human-sized ant is \( 10^6 = 1,000,000 \) times as heavy. That’s 100 times more weight than the human-size ant legs can support! Said another way, a normal ant is 100 times stronger, relative to its weight, than a human (or an ant-sized human).

We can express this result more generally in an equation in which we compare strength and weight. We’ll define a quantity called the relative strength:

\[
\text{Relative Strength} = \frac{\text{Bone or Muscle Strength}}{\text{Weight}}
\]
Since bone or muscle strength scale as the square of the scale factor and weight scales as the cube of the scale factor, we can say that

\[
\text{Relative Strength } \propto \frac{k^2}{k^3} = \frac{1}{k}
\]

The relative strength is inversely proportional to the scale factor. Therefore, if the relative strength of a typical ant is 1, that of a typical human is 1/100. Note that we’re not saying that humans are inherently weaker than ants. We’re simply saying that the size of the bones and muscles doesn’t scale up in the same way as weight.

**Example Problem 6.** How big would the legs of a human-sized ant have to be in order to support the weight of this giant ant?

*Solution.* The relative strength of the human-sized ant would have to be 1 rather than 1/100. We would have to increase the bone strength by a factor of 100. That can be done by increasing the cross-sectional area of the bones by the same factor. With 100 times greater cross-sectional area, how many times larger in radius (or diameter) would a bone be?

Since cross-sectional area increases as the square of the scale factor, then an area that is 100 times greater requires a scale factor of the square root of 100, which equals 10. A human leg bone would therefore have to be 10 times greater in diameter in order that the human have the same strength, in relation to his/her weight, as an ant! The size of the muscles would scale up in a similar way. This would certainly be a very unusual human.

This argument is, of course, a bit oversimplified. The diameter of the bones of the leg isn’t the same throughout the length of the bone. Moreover, the upper leg has one bone, while the lower leg has 2. Nevertheless, the argument that the diameter of the bones, on the average, would have to be much greater in order to support a human-sized ant, is valid.