

## The Practice of Error Analysis<sup>1</sup>

Note that *error* is an unavoidable feature of any experiment. While error can be reduced depending on the technique and measuring instruments, it can't be eliminated. Hence, one should not equate errors with such things as mistakes in calculations or in application of formulas.

### Significant figures (a first approximation to error analysis)

- When multiplying measurements, the product has the same number of significant figures as the measurement having the lesser number of significant figures.
  - When adding measurements, the sum has the same number of decimal digits as the measurement having the lesser number of decimal digits.
  - The last significant figure of a measurement is expected to have some uncertainty.
  - One must take special note of when to express zeros and when they are significant. They are not significant when used simply as placeholders. They are significant whenever they are necessary to indicate a measured digit.
1. Measure the length and width of the following rectangle to the nearest 0.01 cm using a plastic ruler. That is, read to a tenth of the smallest division. Make each measurement 5 times, alternating between length and width measurements. Use different portions of the ruler for each measurement, but always avoid using the ends of the ruler. Be sure to express significant zeros (for example, 5.40 cm rather than 5.4 cm).



Trial	Length (cm)	Length-Mean  (cm)	Width (cm)	Width-Mean  (cm)
1				
2				
3				
4				
5				
Means				

Calculate the mean of each set of measurements. Then calculate the absolute value of the difference between each measurement and the mean. This is termed the deviation. Finally,

---

<sup>1</sup> A Useful Reference: C.E. Swartz, *Used Math*, AAPT, College Park, 1993, Ch. 1.

calculate the mean of the deviations. For the mean, round the results to the appropriate number of decimal digits as per the guidelines given previously. For the mean of the deviations, it doesn't make sense to keep more than one significant figure.

The deviation is a measure of the reproducibility or precision of your measurement technique. It's possible that your measurements are quite reproducible even with an inaccurate technique. For example, you might make a similar error in each measurement. Consistently starting or stopping a stopwatch too early or too late is an example. Such errors are called systematic and can be difficult to detect. One way of detecting them is to do an experiment using two different methods. Another way is to have two people use the same method and compare results.

- Using the mean values of length and width, calculate the area of the rectangle. Give the unrounded value followed by the value rounded to the proper number of significant figures.

Area (unrounded) =

Area (rounded) =

- Calculate the perimeter of the rectangle. Again, give the unrounded and rounded values.

Perimeter (unrounded) =

Perimeter (rounded) =

**Absolute and Percentage Uncertainties in Measurements** (a better approximation to error analysis)

- Estimate the absolute uncertainty in the length and the width. These estimates reflect your own judgment of how close you were able to measure. This can be affected by such things as the fineness of the divisions on the ruler and how well defined the ends of the object being measured are. You may find that your estimates are close to the deviations that you calculated above. In cases where you don't have a set of repeated measurements, estimating the absolute uncertainties may be the only method you have of assessing error.

After estimating the absolute uncertainties, calculate the percentage uncertainties as a ratio of the absolute uncertainty to the mean value of the measurement. Of course, multiply the result by 100.

Measurement	Absolute Uncertainty (cm)	Percentage Uncertainty
Length		
Width		

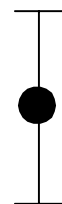
- Calculate the combined absolute and percentage uncertainties in your area calculation. Use the rule that for multiplication (or division), the percentage uncertainty in the product (or

quotient) is the sum of the percentage uncertainties of the measurements being multiplied (or divided). You may want to ask the teacher to justify this rule.

6. Calculate the absolute and percentage uncertainties in your perimeter calculation. Use the rule that for addition (or subtraction), the absolute uncertainty in the sum (or difference) is the sum of the absolute uncertainties of the measurements being added (or subtracted).

Note that the rules for combining uncertainties always give the largest uncertainty. The rules don't take into account the possibility that one of the measurements could be high and another low, thus causing errors to partially cancel.

Uncertainties must be displayed when plotting graphs of measured values. For the ordinate, a line equal in length to the absolute uncertainty is extended above and below the data point and capped as shown in the diagram. For the abscissa, the line is extended to the right and left of the data point. Sometimes, the uncertainties in the abscissa and ordinate are combined and displayed only as vertical error bars. In cases where the error bars would be quite small, the size of the point protector is chosen to represent the span of the error bars. This can be stated in the text that accompanies the graph.



### **Standard Deviation** (a third approximation to error analysis)

You can use your calculator to find the standard deviation of a set of repeated measurements. This method is only justified if there are many measurement trials and if the measurements are expected to cluster around a mean in a Gaussian (bell-shaped) distribution. We will rarely if ever have need of the standard deviation in our experiments.

## Comparing the result of a measurement to a standard

Sometimes we have occasion to compare a measured result to a standard or accepted value obtained with a method that we have reason to believe is superior. For example, after measuring the gravitational field with your pendulum, you could compare it to a value in a reference book. For such comparisons, one generally uses the formula:

$$\frac{|\text{Accepted Value} - \text{Measured Value}|}{\text{Accepted Value}}$$

Multiply by 100 for a percentage. Use of the above formula is often overemphasized in classroom situations where students are expected to get within a certain percentage of an accepted value. A much better scientific practice is to use the method described earlier of finding absolute and percentage uncertainties. In research situations, scientists may try to improve on existing methods of measuring a fundamental constant. They must assess their errors carefully in order to convince their colleagues that they have actually made improvements.

If you simply want to compare two values--for example, initial and final momenta in a collision experiment--and have no reason to expect that one value is better than another, use the following formula:

$$\frac{\frac{1}{2} \cdot (\text{Difference of the 2 Values})}{\frac{1}{2} \cdot (\text{Sum of the 2 Values})}$$

That is, we split the difference and divide by the average. Of course, you can ignore the factors of  $\frac{1}{2}$  in the calculation, since they divide out. Since this calculation doesn't use the absolute value function, you can keep track of which value is the larger of the two. This may help to identify systematic errors in your method.

A common mistake in calculating percentage differences is to overstate the number of digits in the result. Suppose you are calculating the percentage difference between a measured value of  $g$  of 9.85 N/kg and a textbook value of 9.80 N/kg. The mistake is to say that the percentage difference has 3 significant figures, since the individual measurements have that many. But remember that you must calculate the difference of 9.85 and 9.80 in finding the result. By the addition (subtraction) rule, one obtains a number with 1 significant figure, namely 0.05. Hence, the percentage difference will have only 1 significant figure.

## Qualitative Error Assessment

It isn't always possible to evaluate errors quantitatively. For example, you might guess that air friction contributes to error in an experiment with falling objects, but you may not know how to estimate uncertainties. You can, however, note whether you expect air friction to increase or decrease measured values. This helps you to identify whether air friction is a plausible source of error. You would discuss such qualitative errors *and* their expected effects in your conclusion.