Velocity-Dependent Drag Forces, Part II

Introduction: In their second experiment on drag forces, the NCSSM Physics II class—still on board the Flying Unicorn—plans to subject the sphere to a gravitational force in the opposite direction of the drag force. Xircon says, "Let’s just cruise over to my home planet and go into orbit. We have plenty of gravity." Jayne, more than a little skeptical of Xircon’s plan, remarks, "You silly Zirt, the apparent weight of the sphere would be zero in orbit. The sphere and the fluid would be falling at the same rate, so the sphere wouldn’t really be falling through the fluid. We need some other way to simulate a gravitational field. Any ideas?"

Zxyz spoke up, "Simple…we just ask the crew to rotate the Unicorn at a constant rate about a longitudinal axis through its center. The outer wall of the cylindrical ship will then exert an inward normal force on a person or other object standing on the wall. (Zxyz proceeds to draw a diagram like the one to the right.) I’ve read about how, in the last century, Earthbound NCSSM students did a lab in which they rode on the inside of large wheels that turned them upside down at the top. The wheel rotated fast enough that they actually experienced a downward normal force while upside down. Of course, as non-inertial observers, they would feel like they were being pushed radially outward. In like manner, we would feel a force pulling us toward the wall of the rotating ship. To us, it would be the same as if a local gravitational field were acting. Remember the radial icicle problem? Well, it's the same thing. Check out the Equivalence Principle of General Relativity if you don’t believe me."

Mr. Teecher saw an opportunity to reinforce and extend what Zxyz was saying. "Good thinking, Zxyz! Now class, knowing that the radius of the Unicorn is 1000, what would its period of rotation have to be in order to produce a simulated gravitational field of 16.4 \( \ddot{g} \) (equal to the Earth’s gravitational field) at the wall? Give me your answer in guts. All the students immediately accessed their TCIs. "What’s going on here?!" Mr. Teecher exclaimed in disbelief. "You don’t need your TCIs; just use your heads!" SmaarTaX, always the troublemaker, pointed out, "But Teacher Teecher, our TCIs are implanted in our heads." Mr. Teecher ignored this remark and asked for the answer to the problem. Several students responded with the number 24. "That would be the answer in snarks," Mr. Teecher said derisively, "I asked for guts." After some hurried calculations, the correct answer of 49.0 \( \ddot{g} \) was produced. Mr. Teecher then called the ship’s bridge over his PCI, "Captain, crank it up to 128 \( \ddot{g} \), but take it slow so we don’t lose our lunches." The Captain replied, "Roger, Teecher, and good luck on your experiment…Engage."

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1 In the year 2048, NCSSM accepts applicants from other star systems.
2 A gut is the galactic unit of time and is denoted by the symbol \( \ddot{g} \).
3 Texas Computer Implant
4 Personal Communicator Implant
5 NCSSM’s food service provides freeze-dried peanut butter sandwiches for students taking field trips on the Flying Unicorn.
Attention Earthbound Students! Engage your physics faculties and complete the following in your 20th-century recording tablets.⁶

0. Determine the centripetal acceleration at the inner wall of the Unicorn in units of \( \frac{\text{m/s}^2}{\xi} \). Note that \( \xi \) is the symbol for snard.

A metallic sphere is dropped \((v_0 = 0)\) into a tall container of liquid at time \( t = 0 \). The forces acting on the sphere as it falls through the liquid are shown in the diagram. The magnitude of the drag force has the same form as in the previous problem, \( F_d = kv \). (See Velocity-Dependent Drag Forces, Part I). The sphere experiences a second upward force, a buoyant force, whose magnitude is given by \( F_b = mg \), where \( m_f \) is the mass of the fluid displaced by the sphere.

1. Write the net force equation for the sphere and solve for its acceleration.

2. Note that the quantity \( g(1 - m_f/m) \), which should appear in your answer to 1, can also be expressed in terms of the density of the liquid, \( \rho_f \), and the density of the sphere, \( \rho \), as follows: \( g(1 - \rho_f/\rho) \). In effect, the buoyant force can be thought of as reducing the effect of the gravitational field by the amount \((\rho_f/\rho)g\). In order to save some writing, we’ll rename the quantity \( g(1 - \rho_f/\rho) \) with the single symbol, \( g_o \), and note that it is the acceleration of the sphere at \( t = 0 \). Express the acceleration of the sphere at any time, \( t \), in terms of \( g_o \), \( k \), \( m \), and \( v(t) \).

3. Now set \( a = \frac{dv}{dt} \), separate variables, and integrate to find velocity of the sphere as a function of time. (What will your limits be?) The following integral form should help:

\[
\int \frac{dv}{1 + bv} = \frac{1}{b} \ln(1 + bv), \text{ where } b \text{ is a constant.}
\]

4. Solve your result from item 3 for \( v(t) \) and express in simplest form. Check your expression to see that it gives the expected result at \( t = 0 \).

5. Obtain an expression for the terminal velocity, \( v_t \), of the sphere in terms of \( m \), \( g_o \), and \( k \). Then rewrite your equation for \( v(t) \) in terms of \( v_t \). Sketch a \( v-t \) graph of the motion of the sphere, showing the \( v_t \) asymptote.

6. Integrate \( v(t) \) to find \( x(t) \). Check your expression as usual for special cases and sketch a graph.

7. The constant, \( k \), is obtained from the theory of fluid dynamics and is given by \( k = 6\pi \eta r \), where \( \eta \) is the viscosity of the fluid and \( r \) is the radius of the sphere. In the next lab, you’ll drop a small metal sphere into glycerol. You’ll need to know the viscosity of glycerol at the temperature of the fluid. Using the data in the table to the right, interpolate to find the viscosity at 22 °C. Show your work. If you do a curve fit, give the fit equation.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Viscosity (Pa·s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
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</tr>
<tr>
<td>20</td>
<td>1.490</td>
</tr>
<tr>
<td>25</td>
<td>0.954</td>
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<tr>
<td>30</td>
<td>0.629</td>
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⁶ Lab books