Some Review Problems Using Unit Vectors

1. The displacement of a certain particle, \( p \), as a function of time, as measured from a coordinate system, \( m \), is given by: \( \vec{r}_{pm}(t) = \hat{i} + bt\hat{j} + ctk \), where \( b \) and \( c \) represent constants.

   a) Find the velocity and the acceleration of the particle as a function of time as observed from frame \( m \).

   b) A second coordinate system, \( n \), is moving relative to \( m \). From \( n \), the displacement of the same particle as a function of time is found to be: \( \vec{r}_{pn}(t) = \hat{i} + (bt^2 - dt)\hat{j} \), where \( d \) is another constant. Find \( \vec{v}_{nm} \), the velocity of coordinate system, \( n \), relative to \( m \).

   c) Is coordinate system, \( n \), inertial? Explain, making clear that you understand the meaning of inertial.

2. A projectile is fired from ground level on Planet X with an initial velocity of \( 10\hat{i} + 10\hat{j} \) m/s. Three seconds later, while the projectile is still in the air, its velocity is measured to be \( 10\hat{i} - 5\hat{j} \) m/s. Answer the following questions about the projectile.

   a) How far did the projectile travel horizontally in the 3 seconds?

   b) At the 3 second mark, how high is the projectile above its initial position? (Recommended method of solution is to use the average velocity.)

   c) Why don't you need to know the gravitational field strength on Planet X in order to solve parts a and b? Give a different reason for each part. A short phrase can be sufficient for each answer.

3. Use unit vector notation to express the vectors in the following problem. Take the +x-axis in the direction of 3 o’clock and the +y-axis in the direction of 12 o’clock.

   a) Determine the velocity vector of the tip of the second hand of a clock when i) in the 12 o’clock position and ii) in the 1 o’clock position. The radius of the hand’s path is 0.10 m and its period is, of course, 60 s.

   b) Determine the magnitude of the average acceleration of the tip of the second hand between the two positions above.

   c) Determine the magnitude of the instantaneous acceleration of the tip of second hand at any point on the circular path.

   d) How could this problem be changed so that the percentage difference between the average and instantaneous accelerations would be smaller?