

Algebra I
State Mathematics Contest Finals, May 2, 2002

1. Solve for x , given $y = x^2 - 1$ and $x = 2y + 1$.

- a) $x \in \{0, 1\}$ b) $x \in \left\{-\frac{1}{2}, -\frac{3}{4}\right\}$ c) $x \in \left\{-\frac{1}{2}, 1\right\}$ d) $x \in \left\{0, -\frac{3}{4}\right\}$ e) none of these

2. George took three 100 point tests. The average of these three tests was 88. On the second test he missed half as many points as on the first test. His third test was 4 points higher than the second test. What was the grade on the third test?

- a) 88 b) 90 c) 98 d) 94 e) none of these

3. If $C = \frac{x-a}{x-b}$ then

- a) $x = \frac{bC-a}{C-1}$ b) $x = \frac{C-a}{C-b}$ c) $x = \frac{C+a}{C+b}$ d) $x = \frac{1-C}{a-bC}$ e) $x = \frac{C-b}{C-a}$

4. Given the first 10 values of a sequence $\{1, 3, 4, 4, 5, 7, 8, 8, 9, 11, \dots\}$ find the sum of the sixteenth and seventeenth terms.

- a) 30 b) 33 c) 16 d) 35 e) none of these

5. Find the value for x so that the three points, $\{(2, 7), (6, 1), (x, 0)\}$ are collinear.

- a) 7 b) $4\frac{1}{2}$ c) 10 d) $6\frac{2}{3}$ e) none of these

6. The cubic polynomial, $x^3 + 2x^2 - 19x + 12$, has three roots. Their product is:

- a) -2 b) -12 c) 12 d) $-3\sqrt{19}$ e) none of these

7. A right triangle with integer sides has area equal to 30. What is the length of its hypotenuse?
- a) 10 b) 12 c) 13 d) 15 e) none of these
8. Let $f(x) = x^2 + x - 6$. For what values of t does $f(t - 5) = 0$?
- a) -3 and 2 b) -2 and 3 c) 5 d) 2 and 7 e) none of these
9. If x and y are non-negative, $y \leq 6 - x$ and $y \geq x + 4$ then find the largest possible value of $2x + 3y$.
- a) 42 b) 17 c) 24 d) 132.5 e) 18
10. The product of three consecutive positive integers is eight times their sum. Find the value of the middle number.
- a) 0 b) 2 c) 3 d) 4 e) none of these
11. Starting at 12:00 noon how long will it take for the minute hand to make a right angle with the hour hand?
- a) 15 minutes b) 18 minutes c) $16\frac{4}{11}$ minutes d) $16\frac{11}{60}$ minutes e) $16\frac{37}{60}$ minutes
12. Given: $3x - 4y = 7$ and $x + cy = 13$, for what value of " c " will the two equations not have a solution.
- a) $\frac{3}{4}$ b) $\frac{4}{3}$ c) -4 d) $\frac{-4}{3}$ e) none of these
13. Solve for x . $2^{x+1} = 8^x$
- a) 1 b) 3 c) $\frac{-1}{3}$ d) $\frac{1}{2}$ e) none of these

28. Given $(x^2 - 5x + 5)^2 - 1 = 0$ Find the difference between the largest and smallest solution to this equation.

- a) 3 b) 4 c) $2 + \sqrt{5} + \sqrt{3}$ d) $\sqrt{5}$ e) none of these

29. Given $x^2 = 3^8$. Find x .

- a) 9 b) 27 c) 81 d) $27\sqrt{3}$ e) none of these

30. Let $xy = 13$ and $x - y = 7$, find the value of $x^2 + y^2$.

- a) 33 b) 23 c) 120 d) 75 e) 49

31. Define the operation \otimes as $x \otimes y = xy(x - y)$. Find x where $x \neq 0$ and $x \otimes 7 = x$.

- a) $1\frac{1}{7}$ b) $7\frac{1}{7}$ c) 7 d) $\frac{1}{7}$ e) equation has no solution

32. Define the operation \otimes as in the previous problem, $x \otimes y = xy(x - y)$. Which of the following expressions is equivalent to $x \otimes (a + b)$.

- a) $(x \otimes a) + (x \otimes b)$ b) $(x \otimes a) - (x \otimes b)$ c) $(x \otimes a) + (x \otimes b) - 2abx$
d) $(x \otimes a) - a + (x \otimes b) - b$ e) $b(x \otimes a) + a(x \otimes b)$

33. Solve for x : $3 = \frac{2^x + 7}{3 - 2^x}$

- a) $\frac{1}{2}$ b) $\log_2 3$ c) $\log_2 \frac{3}{7}$ d) $\frac{-1}{2}$ e) -1

34. Which of the following equations has $\sqrt{c^2 + 1}$ as a solution for x ?

- a) $c = x + \frac{1}{x - c}$ b) $x = c - \frac{1}{x + c}$ c) $c = \sqrt{x - 1}$ d) $c = x - \frac{1}{x + c}$ e) $c = \frac{1}{\sqrt{x - 1}}$

35. Find the equation of a line that divides the line segment connecting points $(1, 3)$ and $(7, -1)$ into halves, and is parallel to the segment that connects points $(1, 3)$ and $(-1, -1)$.
- a) $y = 2x - 7$ b) $y = 2x + 1$ c) $y = -2x + 9$ d) $y = -2x + 5$ e) none of these
36. There are 4 female and 3 male students hoping to go on a trip. If two people are chosen at random from this group what is the probability that one female and one male student will be picked?
- a) $\frac{2}{7}$ b) $\frac{3}{7}$ c) $\frac{1}{2}$ d) $\frac{4}{7}$ e) $\frac{1}{3}$
37. Given the parabola $y = x^2 - 4x + 6$ and the line $y = mx$, for what positive value of m will the line be tangent to the parabola?
- a) -4 b) $-4 + \sqrt{20}$ c) $-4 + 2\sqrt{6}$ d) 1 e) $\sqrt{40}$
38. Find the non-zero solution for x : $8^x = \left(\frac{1}{4}\right)^{x^2}$
- a) $\frac{2}{3}$ b) $\frac{-2}{3}$ c) 32 d) 2 e) $\frac{-3}{2}$
39. If $0 < \frac{x}{y} < 1$ which of the following must be true?
- a) $x < y < 0$ b) $x < 1$ and $y > 1$ c) $0 < xy < y$
- d) $0 < x < y$ e) $1 < \frac{y}{x}$
40. x is twice as large as y and y is 10 units bigger than z and z is 3 times as big as x , what is the value of $x^2 + y^2 + z^2$?
- a) 164 b) 111 c) 324 d) $\frac{4100}{49}$ e) cannot be determined