

GEOMETRY
State Mathematics Finals Contest, May 1, 2003

1. Given the statement "If a pentagon is regular, then it is equiangular," which of the following is true?
- only the conditional is true
 - only the conditional and contrapositive are true
 - only the conditional, converse, and inverse are true
 - the conditional, converse, inverse, and contrapositive are all true
 - none of the statements are true.

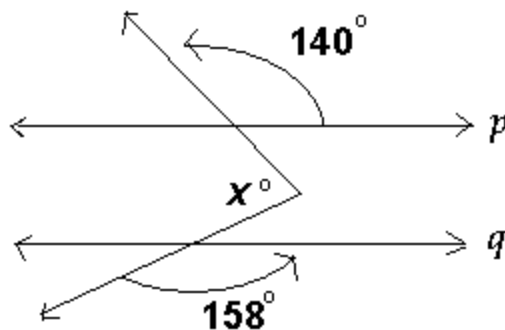
2. \overline{OB} bisects $\angle AOC$. If $m\angle AOB = 2x + 10$ and $m\angle BOC = 8x - 14$, what is $m\angle AOC$?

- 22°
- 25°
- 36°
- 40°
- 44°

3. In $\triangle ABC$, D is a point on \overline{AB} and E is a point on \overline{AC} such that \overline{DE} is parallel to \overline{BC} . If $AD=9$, $BD=6$, and $DE=4$, then $BC=$ _____.

- $2\frac{2}{3}$
- $5\frac{1}{3}$
- $6\frac{2}{3}$
- $7\frac{1}{3}$
- $13\frac{1}{2}$

4. Assume $p \parallel q$ in the figure shown. Then x equals:



- 18
- 22
- 40
- 62
- It cannot be determined from the information given.

5. An isosceles triangle with base 10 cm has an area of 60 cm^2 . Find its perimeter.

- a. 23 cm
- b. 26 cm
- c. 31 cm
- d. 34 cm
- e. 36 cm

6. This figure is formed by three arcs drawn using each vertex of the equilateral triangle as a center and a side of the triangle as a radius. If the shaded area is $18p - 27\sqrt{3}$, then a side of the triangle is

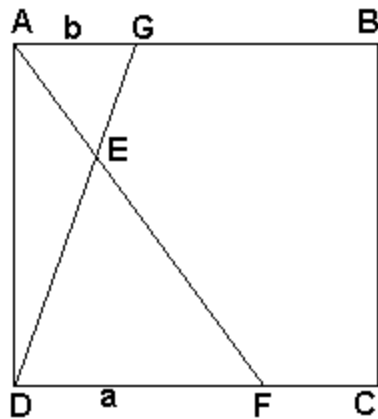


- a. 3
- b. 4
- c. 6
- d. 8
- e. 9

7. Three circles of radii 2, 4, and 6 are tangent to each other externally. Find the area of the triangle formed by connecting their centers.

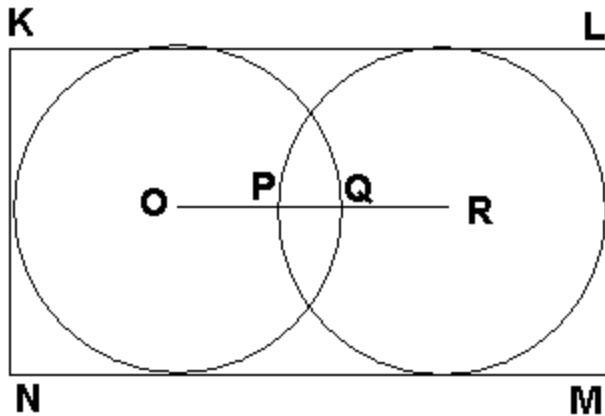
- a. 4
- b. 6
- c. 12
- d. 24
- e. It cannot be determined from the information given.

8. In the unit square, find the distance from E to \overline{AD} in terms of a and b , the lengths of \overline{DF} and \overline{AG} , respectively.

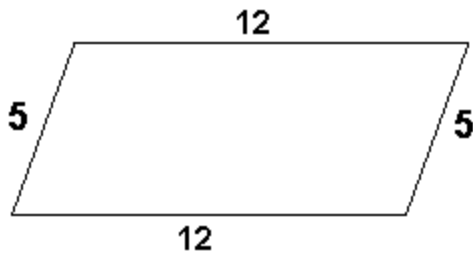


- a. $\frac{ab}{a+b}$
- b. $\frac{b}{a+b}$
- c. $\frac{a-b}{a+b}$
- d. $\frac{a}{a+b}$
- e. $\frac{2a-b}{a+b}$

9. In the figure shown below the circles with centers O and R each have a radius of 2. If $PQ=1$, then what is the perimeter of rectangle $KLMN$?

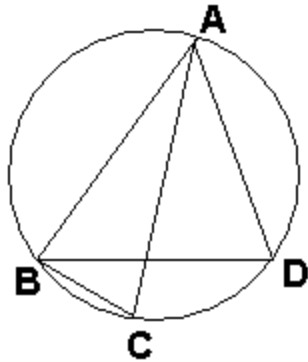


- a. 20 units b. 22 units c. 24 units d. 26 units e. 28 units
10. Tree A stands 5 meters from tree B. A map indicates that treasure is buried 2 meters from tree A and 4 meters from tree B. Assuming the information on the map is true, what is the greatest number of places you would be required to dig (if you know what you are doing) to locate the treasure?
- a. 1
b. 2
c. 3
d. 4
e. It cannot be determined from the information given.
11. The length of the two sides in a parallelogram are 5 cm and 12 cm. Find the sum of the squares of the diagonals of the parallelogram.



- a. 288 cm^2
b. 300 cm^2
c. 338 cm^2
d. 360 cm^2
e. 392 cm^2

12. Points $A, B, C,$ and D lie on a circle with $AB=4$ and $BC=2$; \overline{AC} is a diameter; and $\angle ABD \cong \angle CBD$. What is BD ?



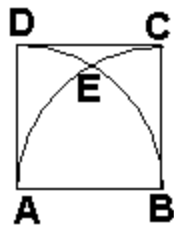
- a. $2\sqrt{3}$
 b. $3\sqrt{2}$
 c. $3\sqrt{3}$
 d. $5\sqrt{2}$
 e. $2\sqrt{2} + 2$
13. The point (a, b) is reflected over the y -axis to the point (c, d) , which is reflected over the x -axis to the point (e, f) . Compute $ab - ef$.
- a. 0
 b. 2
 c. $2ab$
 d. $2cd$
 e. $-2cd$
14. What is the length of the altitude from the right angle to the hypotenuse in a right triangle with legs a and b and hypotenuse c ?
- a. $\frac{ac}{b}$
 b. $\frac{1}{2}ab$
 c. $\frac{bc}{a}$
 d. $\frac{bc}{2}$
 e. $\frac{ab}{c}$

15. A point is randomly selected within the rectangle with vertices $(0,0)$, $(2,0)$, $(2,3)$, and $(0,3)$. What is the probability that the x -coordinate of the point is less than the y -coordinate? Express your answer as a common fraction.
- $1/3$
 - $1/2$
 - $2/3$
 - $7/12$
 - $3/4$
16. What is the distance between the centroids of any two faces of a regular tetrahedron of edge length 1?
- $\sqrt{3}/6$
 - $\sqrt{3}/2$
 - $1/3$
 - $1/2$
 - $\sqrt{2}/3$.
17. In circle O , the chords \overline{AB} and \overline{CD} intersect at point E . If $AE=EB$, $CE=4$, and $ED=9$, what is the length of \overline{AB} ?
- 6
 - 8
 - 10
 - 12
 - 13
18. Each angle of a rectangle is bisected. Let P , Q , R , and S be the points of intersection of the pairs of bisectors adjacent to the same side of the rectangle. Then $PQRS$ is a
- rectangle
 - rhombus
 - parallelogram with unequal adjacent sides
 - quadrilateral with no special properties
 - square
19. Two of the exterior angles of a pentagon have measures 75° and 105° . The measures of the remaining three exterior angles have the ratio 3:4:5. The measure of the smallest angle, in degrees, is
- 15
 - 30
 - 40
 - 45
 - 75

20. The length, width, and height of a rectangular solid are in ratio of 3:4:12. If the diagonal of the solid is 39 inches long, what is the length of the longest dimension of the solid?

- a. 12 in.
- b. 48 in.
- c. 36 in.
- d. 72 in.
- e. none of these

21. Inside square $ABCD$ with side of length x , quarter circle arcs with radius x are drawn using A and B as centers. The arcs intersect at a point E inside the square. The distance from E to the side CD is:

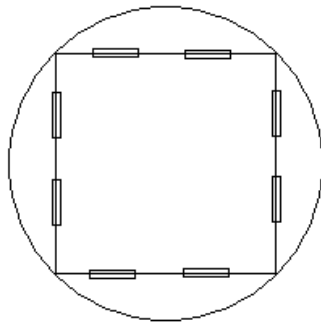


- a. $\frac{x}{2}\sqrt{3}$
- b. $\frac{x}{2}(1 + \sqrt{3})$
- c. $\frac{x}{2}(\sqrt{3} - 1)$
- d. $\frac{x}{2}(2 - \sqrt{3})$
- e. none of these

22. If a plane, 12 units from the center of a sphere, intersects the sphere in a circle with radius 9 units, what is the radius of the sphere?

- a. 5 units
- b. 5.20 units
- c. 7.94 units
- d. 15 units
- e. 16.5 units

23. A round table can be made square by dropping the four leaves down. If a side of the square table measures 36 inches, approximately how much smaller is the area of the table when the leaves are down than when the leaves are up?

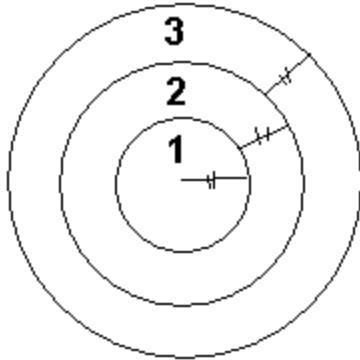


- a. 750 in^2
- b. 850 in^2
- c. 1000 in^2
- d. 1250 in^2
- e. 1300 in^2

24. A circle with center $(4,-2)$ passes through $(7,2)$. Which of the following is the equation of the line tangent to the circle at $(7,2)$?

- a. $4x + 3y = 21$
- b. $2x + 3y = 12$
- c. $4x + 3y = 41$
- d. $3x + 4y = 29$
- e. $3x + 4y = 49$

25. Assume in the game of darts that a beginner hits the dartboard and that the probability of hitting each point on the dartboard is equally likely. What is the probability that a beginner hits the area marked 3?

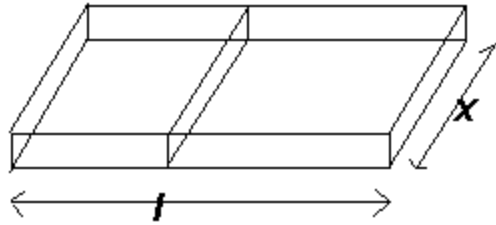


- a. $1/3$
- b. $5/9$
- c. $2/3$
- d. $9/14$
- e. $9/16$

26. The angle of elevation to the top of a radio antenna on top of a building is 53.4° . After moving 200 feet closer to the building, the angle of elevation is 64.3° . Find the height, to the nearest foot, of the building if the height of the antenna is 180 feet.

- a. 368
- b. 256
- c. 412
- d. 506
- e. 585

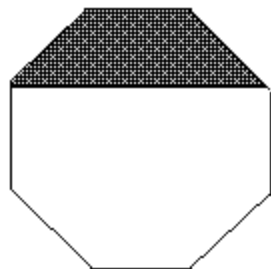
27. A veterinarian wishes to use 132 feet of chain link fencing to enclose a rectangular region and subdivide the region into two smaller rectangles, as shown in the figure. If the total enclosed area is 576 ft^2 , then there are two possible values for x , measured in feet. The difference of these values is



- a. 10 ft b. 20 ft c. 25 ft d. 30 ft e. 36 ft
28. A , B , and C are the vertices of an isosceles right triangle with right angle at B . The area of $\triangle ABC$ is 1. Point E lies on side \overline{AC} of this triangle such that $\overline{AE} \cong \overline{AB}$. Point D lies on side \overline{BC} of this triangle such that $\overline{DE} \perp \overline{AC}$. The area of $\triangle ADC$ is

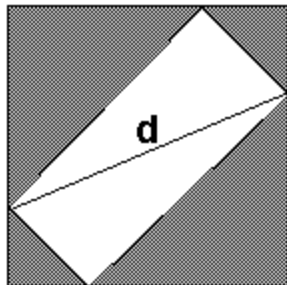
- a. $1/2$
 b. $\sqrt{2}$
 c. $1/\sqrt{2}$
 d. $2-\sqrt{2}$
 e. $\sqrt{2}-1$
29. A 3 by 3 by 3 cube has three square holes, each with a 1 by 1 cross-section running from the center of each face to the center of the opposite face. The total surface area (in square units) of the resulting solid is:
- a. 24
 b. 48
 c. 72
 d. 78
 e. 84

30. The figure below is a regular octagon. What fraction of its area is shaded?



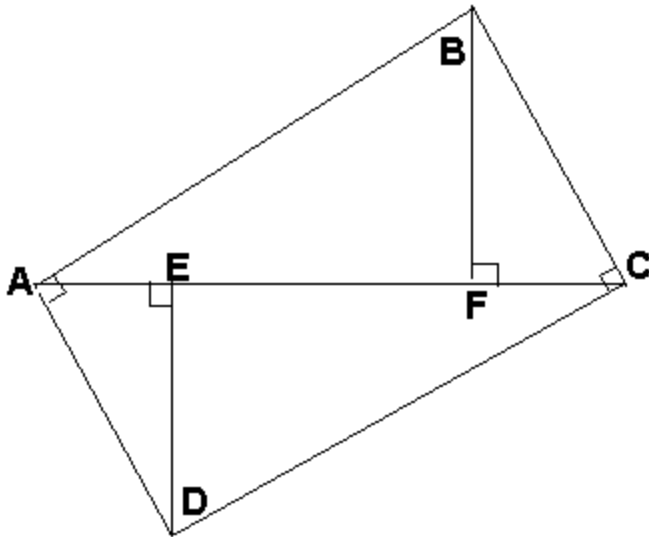
- a. $1/3$ b. $1/4$ c. $1/5$ d. $1/6$ e. $3/8$

31. The sides of a triangle have lengths, 9, 13 and k , where k is an integer. For how many values of k is the triangle obtuse?
- 11
 - 12
 - 15
 - 17
 - 18
32. Four children are arguing over a broken toy. Ali says Barbara broke it. Barbara says Tyler broke it. Tyler and Hei-Lam say they do not know who broke it. Only the guilty child was lying. The child who broke the toy was:
- Ali
 - Barbara
 - Tyler
 - Hei-Lam
 - It cannot be determined from the information given
33. An isosceles right triangle is removed from each corner of a square piece of paper, so that a rectangle remains. The removed triangles are shown as gray in the picture below. Find the length of the diagonal d , if the sum of the areas of the triangles cut off is 200 square units.



- $12\sqrt{2}$
 - $14\sqrt{2}$
 - 18
 - 20
 - 24
34. At 3:00 the minute hand and the hour hand are perpendicular. At this time the second hand is aligned with the minute hand over the 12 on the clock. Call this position 0 seconds. At the next moment when the minute hand and the hour hand are again perpendicular, the position of the second hand is closest to the position:
- 8 seconds
 - 11 seconds
 - 43 seconds
 - 44 seconds
 - 48 seconds

35. $ABCD$ is a quadrilateral with right angles at A and C . Points E and F are on the diagonal \overline{AC} such that \overline{DE} and \overline{BF} are both perpendicular to \overline{AC} . If $AE=3$, $DE=5$, $CE=7$, then the length of BF is:



- a. 3.6 b. 4 c. 4.2 d. 4.5 e. 4.8
36. A chord which is the perpendicular bisector of a radius of length 12 in a circle, has length
- a. $3\sqrt{3}$
b. 27
c. $6\sqrt{3}$
d. $12\sqrt{3}$
e. none of these
37. In $\triangle ABC$ with right angle at C , altitude \overline{CH} and median \overline{CM} trisect the right angle. If the area of $\triangle CHM$ is K , then the area of $\triangle ABC$ is
- a. $6K$
b. $4\sqrt{3}K$
c. $3\sqrt{3}K$
d. $3K$
e. $4K$

38. In $\triangle ADE$, $\angle ADE = 140^\circ$, points B and C lie on the sides of \overline{AD} and \overline{AE} , respectively, and points A, B, C, D, E are distinct. If lengths AB, BC, CD and DE are all equal, then the measure of $\angle EAD$ is

- a. 5°
- b. 6°
- c. 7.5°
- d. 8°
- e. 10°

39. A circle with area A_1 is contained in the interior of a larger circle with area $A_1 + A_2$. If the radius of the larger circle is 3, and if $A_1, A_2, A_1 + A_2$ is an arithmetic progression, then the radius of the smaller circle is

- a. $\sqrt{3}/2$
- b. 1
- c. $2/\sqrt{3}$
- d. $3/2$
- e. $\sqrt{3}$

40. A vertical line divides the triangle with vertices $(0,0)$, $(1,1)$, and $(9,1)$ in the xy -plane into two regions of equal area. The equation of the line is $x =$

- a. 2.5
- b. 3.0
- c. 3.5
- d. 4.0
- e. 4.5