

## Part I: Multiple Choice (20 Problems)

1. When  $\frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}$  is decomposed into partial fractions, with each term reduced to lowest terms, the sum of the numerators is

- a. 16      b. 15      c. 14      d. 12      e. -4

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2. A person deposits \$500 into a savings account at the end of every month for 4 years at 6% annual rate compounded monthly. How much interest will be earned during the 4 years?

- a. \$1440      b. \$1480.27      c. \$2024.39      d. \$3048.92      e. \$4098.46

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3. There are 100 members of the senate, 2 from each state. In how many ways can a committee of 5 senators be formed if no state may be represented more than once?

- a. 2,118,760      b. 75,287,520      c. 4,950      d. 67,800,320      e. 254,251,200

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4. You have 6 sticks of lengths 10, 20, 30, 40, 50, and 60 centimeters. The number of non-congruent triangles that can be formed by choosing three of the sticks to make the sides is

- a. 3      b. 6      c. 7      d. 10      e. 12

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5. A glass box 7 cm  $\times$  12 cm  $\times$  18 cm, closed on all six sides is partly filled with colored water. When the box is placed on one of its 7  $\times$  12 sides the water level is 15 cm above the table. When the box is placed on one of its 7  $\times$  18 sides the water level above the table, in centimeters, will be

- a. 7.5      b. 9      c. 10      d. 12.5      e. none of these

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6. Two integers are said to be partners if both are divisible by the same set of prime numbers. The number of positive integers less than 25 that have no partners less than 25 is

- a. 11      b. 12      c. 13      d. 16      e. 24

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7. There are four cottages on a straight road. The distance between Ted's and Alice's cottages is 3 kilometers. Both Bob's and Carol's cottages are twice as far from Alice's as they are from Ted's. In kilometers, the distance between Bob's and Carol's cottages is

- a. 1      b. 2      c. 3      d. 4      e. 6

8. Al, Bee, Cecil, and Di have \$16, \$24, \$32, and \$48 respectively. Their father proposed that Al and Bee share their wealth equally, and then Bee and Cecil do likewise and then Cecil and Di. Their mother's plan is the same except that Di and Cecil begin by sharing equally, then Cecil and Bee and then Bee and Al. The number of children who end up with more money under their father's plan than under their mother's is

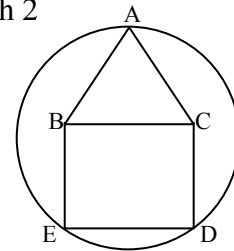
- a. 0            b. 1            c. 2            d. 3            e. 4

9. Let  $t_0 = 2004$  and recursively define  $t_{k+1} = \lfloor \frac{1}{2} (t_0 - t_1 - t_2 - \dots - t_k) \rfloor$  where  $\lfloor x \rfloor$  is the greatest integer less than or equal to  $x$ . Find the least number  $k$  so that  $t_k = 0$ .

- a. 9            b. 10            c. 11            d. 12            e.  $t_k$  is never 0

10. A pentagon is made up of an equilateral triangle ABC of side length 2 on top of a square BCDE. Circumscribe a circle through points A, D and E. The radius of the circle is:

- a)  $\frac{\sqrt{3}}{2} + 1$     b) 2    c)  $\sqrt{3} + 1$     d)  $5 - 2\sqrt{3}$     e)  $\sqrt{2}$



11. Two of the roots of the equation  $2x^3 - 3x^2 + px + q = 0$  are 3 and  $-2$ . The third root is

- a.  $\frac{1}{2}$             b.  $\frac{-5}{2}$             c.  $-3$             d.  $\frac{1}{3}$             e. 1

12. How long is the side of the largest equilateral triangle that can be inscribed in a square whose side has length 1?

- a. 1            b.  $\frac{\sqrt{5}}{2}$             c.  $\frac{3\sqrt{5}}{4}$             d.  $2 - \sqrt{3}$             e.  $\sqrt{8 - 4\sqrt{3}}$

13. A round table can be made square by dropping the four leaves. If a side of the square table measures 36 inches, approximately how much smaller is the area of the table when the leaves are down than when the leaves are up?

- a.  $750 \text{ in}^2$     b.  $850 \text{ in}^2$     c.  $1000 \text{ in}^2$     d.  $1250 \text{ in}^2$     e.  $1300 \text{ in}^2$

14. From a fixed point on a circle chords are drawn to the other points on the circle. What is the locus of midpoints of the chords?

- a. non-circular ellipse    b. circle    c. hyperbola    d. parabola    e. line



15. The sum of the two largest numbers  $x$  for which the determinant

$$\begin{vmatrix} 2x-2 & 1 & 4 \\ 6x-11 & 2x-5 & 2x+5 \\ -2x+2 & -1 & x-2 \end{vmatrix} \text{ equals zero is}$$

- a. 20            b. 5            c. 2            d.  $\frac{-1}{2}$             e. none of the above

16. Consider the circles with radii  $4\sqrt{5}$  and which are tangent to the line  $x - 2y = 20$  at the point  $(6, -7)$ . The sum of the  $x$  coordinates of the centers of the circles is

- a. 12            b. -14            c. 3            d. -5            e. 2

17. Given the equation  $x^3 - 2x^2 + x - 3 = 0$ , an equation whose roots are each 2 less than the roots of the given equation is

- a.  $x^3 - 8x^2 + 21x - 21 = 0$             b.  $x^3 - 4x^2 - x - 5 = 0$             c.  $x^3 - 4x^2 + 2x - 6 = 0$   
 d.  $x^3 + 4x^2 + 5x - 1 = 0$             e.  $x^3 + 4x^2 - 2x + 6 = 0$

18. An experiment consists of choosing with replacement an integer at random among the numbers from 1 to 9 inclusive. If we let M denote a number that is an integral multiple of 3 and N denote a number that is not an integral multiple of 3, which of the following sequences of results is least likely?

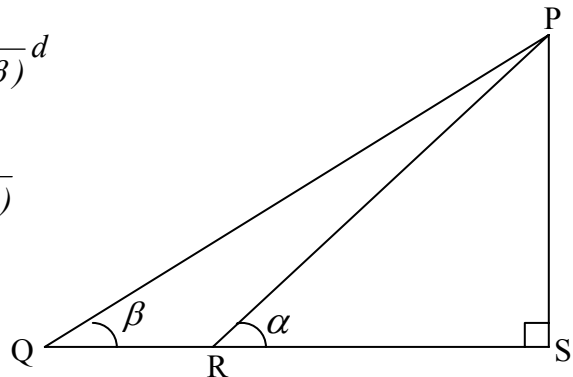
- a. M N N M N    b. N M M N    c. N M M N M    d. N N M N    e. M N M M

19. An 8 foot by 8 foot area has been tiled by one foot square tiles. Two of the tiles were defective. What is the probability that the two defective tiles share an edge?

- a.  $\frac{1}{8}$             b.  $\frac{1}{12}$             c.  $\frac{1}{16}$             d.  $\frac{1}{18}$             e.  $\frac{1}{64}$

20. In the diagram if  $QR = d$ , then PS equals

- a.  $\frac{\sin(\beta)}{\sin(\alpha - \beta)} d$             b.  $\frac{\tan(\beta)}{\tan(\alpha) - \tan(\beta)} d$   
 c.  $\frac{d}{\tan(\alpha) - \tan(\beta)}$             d.  $\frac{d}{\cot(\beta) - \cot(\alpha)}$   
 e.  $\frac{\sin(\alpha) \sin(\beta)}{\cos(\alpha)(\sin(\alpha) - \sin(\beta))} d$



## Part II: Integer Answers (15 Problems)

1. Find  $n$  so that

$$\frac{1}{1+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \dots + \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}} = 100$$

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2. If  $\tan 3x$  is written in terms of  $\tan x$ ,  $\tan 3x = \frac{A \tan x - B \tan^3 x}{1 - C \tan^2 x}$  find  $A + B + C$ .

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3. Consider the equation  $15x + 14y = 7$ . Find the largest four digit integer  $x$  for which there is an integer  $y$  so that the pair  $(x, y)$  is a solution.

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4. Let  $P$  be the set of primes that divide  $200!$  (i.e. 200 factorial). What is the largest integer  $k$ , so that the set of primes that divides  $k!$  is equal to  $P$ ?

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5. What is the remainder when  $7^{348} + 25^{605}$  is divided by 8?

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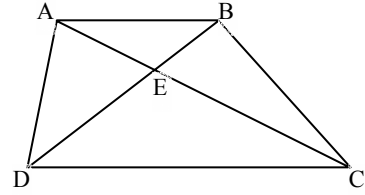
6. How many possible values can there be for three coins selected from among pennies, nickels, dimes and quarters?

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7. A water tank has been sanitized by pouring in chlorine bleach. Bleach is toxic at the level needed to sanitize, so you need to flush out the tank using clean water. The result is that after each hour of flushing there is a 19% reduction in the bleach concentration. Assume that when you began flushing, the bleach concentration is 150 mg/gal. You can safely use the water tank for drinking purposes when the bleach concentration is below 0.7 mg/gal. What is the minimum number of whole hours you should flush the tank for safe drinking purposes?

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8. In a trapezoid ABCD with AB parallel to CD, the diagonals intersect at point E. The area of triangle ABE is 32 and of triangle CDE is 50. Find the area of the trapezoid.



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9. Find the number of 4 digit positive integers which are divisible by 3 and/or 7.

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10. What is the smallest positive integer which when divided by 10, 9, 8, 7, 6 leaves the remainder 9, 8, 7, 6, 5 respectively?

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11. If the product of three numbers in geometric progression is 216 and their sum is 19, then the largest of the three numbers is

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12. Among all collections of positive integers whose sum is 28, what is the largest product that the integers in S can form?

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13. Consider the set S of positive integers  $d$  for which there exists an integer  $n$  such that  $d$  evenly divides both  $(13n+6)$  and  $(12n+5)$ . Then the sum of the elements of S is

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14. Suppose that  $x$  and  $y$  are two real numbers such that  $x - y = 2$  and  $x^2 + y^2 = 8$ . Find  $x^3 - y^3$ .

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15. What is the remainder when  $1! + 2! + 3! + 4! + \dots + 99! + 100!$  is divided by 18?

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