ALGEBRA I
State Mathematics Contest Finals
May 5, 2005

1. If $\frac{x}{2}-\frac{x}{6}$ is an integer, which of the following statements must be true?

   a. x is positive
   b. x is odd
   c. x is even
   d. x is a multiple of 3
   e. x is a multiple of 6

2. Which of the following is equivalent to $7^{77}-7^{76}$?

   a. 7
   b. 7^{77-76}
   c. $7^{77\div 76}$
   d. $7(77-76)$
   e. $7^{76}(6)$

3. In a class of 10 boys and 15 girls, the boys' average score on the final exam was 80 and the girls' average score was 90. What was the average score for the whole class?

   a. 83
   b. 84
   c. 85
   d. 86
   e. 87

4. If $f(x)=3x^2+4$, $f(x+1)=$
   a. $3x^2+5$
   b. $3x^2+x+4$
   c. $3x^2+4$
   d. $3x^2+2x+5$
   e. $3x^2+6x+7$

5. A crow flies (always in a straight line) from its nest to a point 10 miles north and 7 miles east of the nest. At that point, it finds a scarecrow, and so it turns and flies to a point 4 miles further north and 5 miles further east. From there it flies directly back to the nest. To the nearest mile, what is the total distance the crow flies?

   a. 32 miles
   b. 33 miles
   c. 37 miles
   d. 40 miles
   e. 44 miles

6. Let b be a positive number such that the system $\begin{cases} ax + 3y = 1 \\ 5x + ay = b \end{cases}$ has an infinite number of solutions. Then, correct to the nearest hundredth, b equals

   a. 0.60
   b. 1.29
   c. 1.67
   d. 3.87
   e. no unique answer

7. The table below displays data that relates the number of oil changes per 3 year time period and the cost of engine repairs. Fit a linear regression equation to the data and use the equation to predict the cost of engine repairs for 7 oil changes per 3 year time period.

Oil changes per 3 years	3	5	3	1	6	0	10	7
Cost of repairs ($)	300	300	400	700	100	600	0	150

   a. $135.67
   b. $138.63
   c. $145.00
   d. $150.00
   e. $154.33

8. If a is 50% larger than c and b is 25% larger than c, then a is what percent larger than b?

   a. 20%
   b. 25%
   c. 50%
   d. 100%
   e. 200%

9. The cost of living in each quarter (3 months) increased by 2% over the previous quarter. To the nearest tenth of a percent, to what annual percentage rate of increase does this correspond?

   a. 2%
   b. 4.8%
   c. 8%
   d. 8.2%
   e. 16%

10. The units digit of $7^{42} + 42^{7}$ is

   a. 1
   b. 3
   c. 5
   d. 7
   e. 9

11. Find the value of digit *A* if the five-digit number 12*A*3*B* is divisible by both 4 and 9 and $A \neq B$.

a. 1
b. 2
c. 3
d. 6
e. 7

12. In a prehistoric village, rocks, stones, and pebbles were used as money. The relative values of the "coins" were:

$$1\,\text{rock} = 7\,\text{stones}$$
$$1\,\text{rock} = 49\,\text{pebbles}$$

If a man used 6 rocks to purchase a hide that costs 5 rocks, 2 stones, and 3 pebbles, how much change was he owed?

a. 1 rock, 5 stones, 4 pebbles
b. 5 stones, 4 pebbles
c. 4 stones, 4 pebbles
d. 5 stones, 5 pebbles
e. 6 stones, 5 pebbles

13. One root of $mx^2 + 8x + 4 = 0$ is three times the other root. The value of *m* is:

a. -5
b. 5
c. $-\frac{28}{9}$
d. -3
e. 3

14. $3^n + 3^n =$

a. 3^{2n}
b. $2 \cdot 3^n$
c. 6^n
d. 6^{2n}
e. 9^{2n}

15. If $\frac{a-1}{a+1}=\frac{b-4}{b+3}$ then b equals:

a. $\frac{7a+1}{2(1-a)}$

b. $\frac{a+1}{2}$

c. $\frac{2a}{a+1}$

d. $\frac{7a+1}{2}$

e. $\frac{-7a-1}{2}$

16. A rectangle has perimeter 24 inches. If the length of the diagonal of this rectangle is 10 inches, what is the area of the rectangle?

a. 11 square inches
b. 20 square inches
c. 22 square inches
d. 27 square inches
e. 32 square inches

17. In the equation, $4x+y=10$, if an x-value is increased by 3, what would be the effect on the corresponding y-value?

a. The value of y will decrease by 12.
b. The value of y will decrease by 2.
c. The value of y will decrease to half as large.
d. The value of y will increase by 3.
e. The value of y will be 3 times as large.

18. A new automobile is purchased for \$25,000. If $V=25000(0.8)^x$ gives the car's value after x years, then approximately how long will it take for the car to be worth half its purchase price?

a. 2 years
b. 3 years
c. 4 years
d. 5 years
e. 6 years

19. On a television show, a player receives five points for answering an easy question and eleven points for a hard one. What is the largest integer that cannot be a contestant's total score in the game?

    a. 29
    b. 39
    c. 43
    d. 49
    e. 53

20. Given $6^{x+y} = 36$ and $6^{x+5y} = 216$, what is the value of *x*?

    a. 0.25
    b. 0.75
    c. 1.25
    d. 1.50
    e. 1.75

21. Find the sum of the solutions of the equation $8^{x^2+3x+10} = 4^{x^2-x}$.

    a. -2.5
    b. -7
    c. -11
    d. -12.5
    e. -14

22. If $f(x+1) = x^2 + 3x + 5$, then find $f(x)$.

    a. $x^2 + 3x + 4$
    b. $x^2 + x + 3$
    c. $x^2 + 2x + 4$
    d. $x^2 - x + 2$
    e. $x^2 - 2x + 2$

23. Suppose *n* and *x* are positive and let $\langle n \rangle = \frac{2(n^2 + n)}{n+1}$, then $\langle 3x \rangle =$

    a. $6x$
    b. $6(x-1)$
    c. $3(x-1)$
    d. $3xn$
    e. 1

24. If $L@K = L + \frac{K}{L}$, then $L@(L@K)$ equals

a. $L + 2K + \frac{K}{4}$

b. $2L + \frac{K}{4} + 1$

c. L

d. $L + 1 + \frac{K}{L^2}$

e. $\frac{L}{K}$

25. Given that *n* is a positive even integer, $5n + 4$ will always be divisible by:

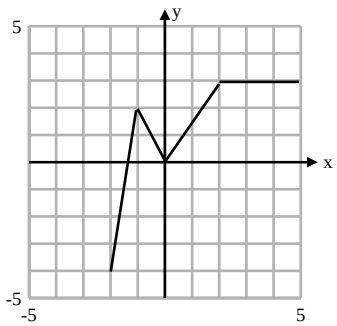
a. 9
b. 4
c. 5
d. $5n$
e. 2

26. In a movie theatre line, *x* people are behind Mark, who is *y* places in front of Sam. If there are *z* people in front of Sam, how many people are in the line?

a. $z - x + y + 2$
b. $z + x - y + 1$
c. $z - x + y - 1$
d. $z + x - y$
e. $z - x + y$

27. A graph of the function *f* is shown below. For which value of *x* does $f(x+3) = f(x) + 1$?

a. 0
b. $\frac{1}{3}$
c. $\frac{2}{3}$
d. 1
e. $\frac{4}{3}$



28. The length of a rectangular pen is one more than twice its width. If both dimensions are increased by 5 units, which expression gives the resulting increase in area?

    a. $15x + 30$
    b. $16x + 30$
    c. $2x^2 + 11x + 30$
    d. $2x^2 + 15x + 30$
    e. $2x^2 + 16x + 30$

29. If the graphs of $4y - x + 3 = 0$ and $3y + ax + 2 = 0$ meet at right angles, then *a* must equal

    a. $\frac{3}{4}$
    b. $-\frac{4}{3}$
    c. -12
    d. 12
    e. none of these

30. Let $a * b$ be a binary operation defined on the integers by $a * b = b - 2$. Under what conditions on integers *a*, *b*, and *c* is * associative?

    a. $a * b = 2$
    b. *a* or *b* or *c* is 2
    c. *a* or *b* or *c* is zero
    d. $a = b = c$
    e. The operation * is never associative

31. Thirty-one books are arranged from left to right in order of increasing prices. The price of each book differs by $2.00 from that of each adjacent book. For the price of the book at the extreme right, a customer can buy the middle book and an adjacent one. Then

    a. The cheapest book sells for $4.00.
    b. The middle book sells for $36.00.
    c. The most expensive book sells for $64.00.
    d. The adjacent book referred to is the one to the right of the middle book.
    e. None of the statements above is true.

32. If $a > b$, then $|a - b| + |b - a|$ is equal to what expression?

    a. 0
    b. $2a$
    c. $2b$
    d. $2a + 2b$
    e. $2a - 2b$

33. The display on a digital clock reads 6:38. What will the clock display twenty-seven digit changes later?

    a. 7:00
    b. 7:01
    c. 7:02
    d. 7:03
    e. 7:04

34. On each day of its life a scroboscopus squares its number of legs. For example, if a scroboscopus had two legs on the first day of its life, it would have 4 legs on the second day of its life, 16 legs on the third day of its life, and so on. Tom bought some newborn scroboscopi from the pet store. Some had 2 legs to start with, some had 3 legs, and some had 5 legs. Before they have a chance to increase their number of legs, the total leg count came out to 58 legs. The next day the total leg count came out as 164. The day after that the total number of legs increased to 1976. How many scroboscopi did Tom buy altogether?

    a. 15
    b. 16
    c. 21
    d. 23
    e. 30

35. During the Christmas season, many postal workers have to work overtime, so a supervisor at one large post office planned a late-night snack for the employees. She ordered 1 extra large pizza for every two workers, 1 large bag of potato chips for every three workers, and 1 two-liter bottle of cola for every four workers. When the order arrived, 26 items were delivered. How many employees were working that evening?

    a. 6
    b. 12
    c. 24
    d. 36
    e. 48

36. If x and y are positive numbers and the average of 4, 20, and x is equal to average of y and 16, then the ratio x:y is
    a. 3:2
    b. 2:3
    c. 1:1
    d. 2:5
    e. 5:2

37. In a dice game, a player rolls two dice. His score is the larger of the two numbers on the dice. For example, if he rolls 3 and 5, his score is 5, and if he rolls 4 and 4, his score is 4. What is the probability that his score is 3 or less?

a. $\frac{1}{4}$
b. $\frac{7}{36}$
c. $\frac{5}{36}$
d. $\frac{1}{3}$
e. $\frac{2}{9}$

38. The River Rafting Company offers a 1-day trip for groups. They charge \$50 per person but have a minimum charge of \$900 and a maximum charge of \$1350 for one raft for the day. Each raft can hold 33 passengers. If each member of a hiking club paid \$45 for a 1-day trip using one raft, how many went rafting?

a. 30
b. 29
c. 28
d. 27
e. 26

39. If $f(2x) = \frac{2}{2+x}$ for all $x > 0$, then $2f(x) =$

a. $\frac{2}{1+x}$
b. $\frac{2}{2+x}$
c. $\frac{4}{1+x}$
d. $\frac{4}{2+x}$
e. $\frac{8}{4+x}$

40. How many ordered pairs (m,n) of positive integers are solutions to $\frac{4}{m}+\frac{2}{n}=1$?

a. 1
b. 2
c. 3
d. 4
e. more than 4