

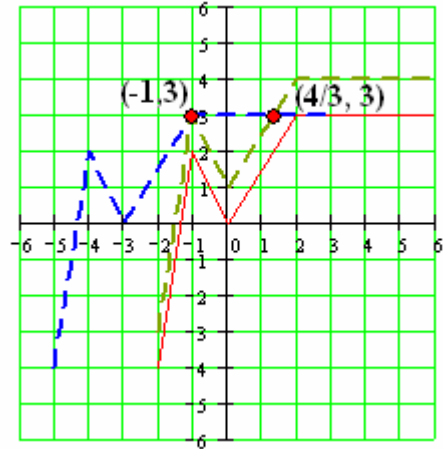
**2005 State Mathematics Finals – Algebra I
Solutions**

1. D. $\frac{x}{2} - \frac{x}{6} = \frac{3x}{6} - \frac{x}{6} = \frac{2x}{6} = \frac{x}{3}$, so x is a multiple of 3.
2. E. $7^{77} - 7^{76} = 7^{76}(7-1) = 7^{76} \cdot 6$.
3. D. Average = $\frac{10 \cdot 80 + 15 \cdot 90}{25} = 86$
4. E. $f(x+1) = 3(x+1)^2 + 4 = 3(x^2 + 2x + 1) + 4 = 3x^2 + 6x + 7$.
5. C. Let x be the first distance, y the second distance, and z the return distance. Using the Pythagorean theorem, $x^2 = 7^2 + 10^2 \Rightarrow x = \sqrt{149}$,
 $y^2 = 4^2 + 5^2 \Rightarrow y = \sqrt{41}$, and $z^2 = 14^2 + 12^2 \Rightarrow z = \sqrt{340}$, so the total distance is
 $x + y + z = \sqrt{149} + \sqrt{41} + \sqrt{340} \approx 37.04$.
6. B. For the system of linear equation to contain infinitely many points, the lines must coincide, which happens when one equation is a scalar multiple of the other, so $ak = 5, 3k = a, k = b$, which implies that $k = \frac{5}{a} = \frac{a}{3} = b$, so
 $a^2 = 15 \Rightarrow a = \sqrt{15}$ and $b = \frac{\sqrt{15}}{3} \approx 1.29$
7. B. The linear regression equation with values rounded to 3 decimal places is $y = -68.616x + 618.946$. When $x = 7, y = -68.616(7) + 618.946 \approx \138.63 .
8. A. $a = 1.5c, b = 1.25c \Rightarrow c = \frac{b}{1.25}$ and $a = 1.5\left(\frac{b}{1.25}\right) = \frac{6}{5}b = 1.2b$, so a is 20% larger than b .
9. D. Let the cost of living at the beginning of the year be x . At the end of the first quarter it is $1.02x$. At the end of the second quarter it increases another 2% to $(1.02)^2 x$. Continuing in this manner, at the end of four quarters the cost of living is $(1.02)^4 x = 1.082x$. Thus, the increase of 2% per quarter corresponds to an annual increase of approximately 8.2%.

10. D. To find the last digit of $7^{42} + 42^7$, we find the last digit of the two quantities in the sum. To find the last digit of 7^{42} , we notice that successive powers of 7 end in 7, 9, 3, 1, and then repeat in groups of 4, making the last digit of 7^{42} 9, since 42 has remainder 2 when divided by 4. Similarly, powers of 2 end in 2, 4, 8, 6, and back to 2, also cycling in groups of 4. Thus the last digit of 42^7 is 8, since the remainder is 3. Thus the sum is the same as $9 + 8$, or 7.
11. A. Since we want 12a3B to be divisible by 9, the sum of the digits, $A + B + 6$, must be divisible by 9. Since this sum cannot equal 0, it must be 9 or 18, since 27 is too high even if A and B are both 9. So $A + B = 3$ or $A + B = 12$. Since we want 12A3B is to be divisible by 4, the number 3B formed by taking the last two digits must be divisible by 4. this can only be is B is 2 or 6. If $B = 6$, then by the restrictions on $A + B$, A must equal -3 (an impossibility) or 6, which is banned because A cannot equal B. This $B = 2$, so $A = 1$, or $A = 10$, another impossibility. The only solution is $A = 1$, $B = 2$.
12. C Let $r =$ the number of rocks, $s =$ the number of stones, p the number of pebbles. so $1r = 7s$ and $1r = 49p \Rightarrow 7s = 49p \Rightarrow s = 7p$, so $6r - (5r + 2s + 3p) = (5r + 7s) - (5r + 2s + 3p) = 5s - 3p$, but we cannot have a negative number of coins, so $5s - 3p = (4s + 7p) - 3p = 4s + 4p$.
13. E. Let a and $3a$ be the roots. Then $ma^2 + 8a + 4 = 0$ and $m(3a)^2 + 8(3a) + 4 = 0$, so $9ma^2 + 72a + 36 = 0$ and $9ma^2 + 24a + 4 = 0$. Subtracting the second from the first gives $48a + 32 = 0$, so $a = -\frac{2}{3}$. Thus $m\left(-\frac{2}{3}\right)^2 + 8\left(-\frac{2}{3}\right) + 4 = 0 \Rightarrow \frac{4}{9}m - \frac{16}{3} + 4 = 0 \Rightarrow \frac{4}{9}m = \frac{4}{3} \Rightarrow m = 3$.
14. B. $3^n + 3^n = 2 \cdot 3^n$
15. D. $\frac{a-1}{a+1} = \frac{b-3}{b+3} \Rightarrow (a-1)(b+3) = (a+1)(b-4)$, so $ab + 3a - b - 3 = ab - 4a + b - 4 \Rightarrow 7a + 1 = 2b \Rightarrow b = \frac{7a+1}{2}$.
16. C. A rectangle with perimeter 24'' has length = width equal to 12''. Let $x =$ width. Then $12 - x = \text{length}$ and $(12 - x)^2 + x^2 = 10^2 \Rightarrow 144 - 24x + x^2 + x^2 = 100$, so $2x^2 - 24x + 44 = 0 \Rightarrow x^2 - 12x + 22 = 0$, so $x(12 - x) = 22$, so Area = 22 sq. in.
17. A. $4x + y = 10 \Rightarrow y = -4x + 10$. If x is increased by 3, then $y' = -4(x+3) + 10 = -4x - 2$, so y is decreased by 12 since $10 - (-2) = 12$.

18. B. $\left(\frac{1}{2}\right)25000 = 25000(0.8)^x \Rightarrow \frac{1}{2} = 0.8^x \Rightarrow x \approx 3.$
19. B The set of possible scores are numbers of the form $5e + 11h$, where e and h are nonnegative integers.
 $e = 0, 5e + 11h = \{0, 11, 22, 33, \dots\}$
 $e = 1, 5e + 11h = \{5, 16, 27, 38, \dots\}$
For $e = 2, 5e + 11h = \{10, 21, 32, 43, 54, \dots\}$ Continuing in this manner, it can be
 $e = 3, 5e + 11h = \{15, 26, 37, 48, 59, \dots\}$
 \vdots
shown that 39 cannot be expressed in the form $5e + 11h$, while the larger choices can. Note $53 = 5 \cdot 4 + 11 \cdot 3, 49 = 5 \cdot 1 + 11 \cdot 4, 43 = 5 \cdot 2 + 11 \cdot 3.$
20. E. $6^{x+y} = 36 \Rightarrow 6^{x+y} = 6^2 \Rightarrow x + y = 2 \Rightarrow y = 2 - x$ and
 $6^{x+5y} = 216 \Rightarrow 6^{x+5y} = 6^3 \Rightarrow x + 5y = 3.$ So
 $x + 5(2 - x) = 3 \Rightarrow x + 10 - 5x = 3 \Rightarrow -4x = -7 \Rightarrow x = 1.75$
21. C. $8^{x^2+3x+10} = 4^{x^2-x} \Rightarrow 2^{3(x^2+3x+10)} = 2^{2(x^2-x)} \Rightarrow 3(x^2 + 3x + 10) = 2(x^2 - x),$ so
 $3x^2 + 9x + 30 = 2x^2 - 2x \Rightarrow x^2 + 11x + 30 = 0 \Rightarrow (x + 5)(x + 6) = 0,$ so $x = -5$ or $-6,$
so the sum is $-5 + (-6) = -11.$ Note that the sum of the roots of the equation
 $x^2 - bx + c = 0$ is always $b,$ so in this case, the sum is -11 without having to find the specific roots.
22. B. Since $x = (x - 1) + 1,$
 $f(x) = f(x - 1 + 1) = (x - 1)^2 + 3(x - 1) + 5 = (x^2 - 2x + 1) + 3x - 3 + 5 = x^2 + x + 3.$
23. A. e for $n \neq -1, \frac{2(n^2 + n)}{n + 1} = \frac{2n(n + 1)}{(n + 1)} = \frac{2n}{1} = 2n,$ so $\langle 3x \rangle = 2 \cdot (3x) = 6x.$
24. D. $L @ (L @ K) = L @ \left(L + \frac{K}{L}\right) = L + \left(\frac{L + \frac{K}{L}}{L}\right) = L + \frac{L}{L} + \frac{K}{L^2} = L + 1 + \frac{K}{L^2}$
25. E. Since n is even, $n = 2k,$ for some integer $k.$ So
 $5n + 4 = 5(2k) + 4 = 2(5k + 2),$ which must be divisible by 2.
26. B. Since Mark is y places in front of Sam, there are $x - y$ people behind Sam, excluding Sam. Then, since there are z people in front of Sam, again excluding Sam, the total number of people in line is $x - y + z + 1.$

27. E. The question asks a graphical question: When (for what x -values) does the function shifted three units to the left equal the function shifted one unit up? The figure shows that $x = -1$ is one such point. The other point of intersection is two-thirds of the way up a segment that has a width of 2. Therefore, its horizontal change is $4/3$. The segment begins at $x = 0$, so $x = 4/3$ is the other solution. We can check numerically.



28. A. The x and $2x+1$ be the width and length, respectively, of the original rectangle. Then the new rectangle has dimensions $x+5$ and $2x+6$. The areas are $x(2x+1) = 2x^2 + x$ and $(x+5)(2x+6) = 2x^2 + 16x + 30$, respectively, so the increase in area is $(2x^2 + 16x + 30) - (2x^2 + x) = 15x + 30$.

29. D. The slopes of the lines must be negative reciprocals.

$$4y - x + 3 = 0 \Rightarrow 4y = x - 3 \Rightarrow y = \frac{1}{4}x - \frac{3}{4} \text{ and}$$

$$3y + ax + 2 = 0 \Rightarrow 3y = -ax - 2 \Rightarrow y = -\frac{a}{3}x - \frac{2}{3}. \text{ So } \frac{1}{4} = -\left(-\frac{3}{a}\right) \Rightarrow a = 12.$$

30. E. If $*$ is associative under some conditions, then $a*(b*c) = (a*b)*c$. Then $a*(b*c) = a*(c-2) = c-2-2 = c-4$ and $(a*b)*c = (b-2)*c = c-2$. Since $c-4 = c-2$, has no solution for c , there is no integer satisfying the desired condition and $*$ is not associative.

31. E. The book prices are the terms of an arithmetic sequence where $a_1 = x$, $a_2 = x+2$, $a_3 = x+2 \cdot 2$, \dots , $a_n = x+(n-1) \cdot 2$. Either $a_{31} = a_{15} + a_{16}$ or $a_{31} = a_{16} + a_{17}$. If $a_{31} = a_{15} + a_{16}$, then $x+60 = (x+30) + (x+28) \Rightarrow x = 2$. If $a_{31} = a_{16} + a_{17}$, then $x+60 = (x+30) + (x+32) \Rightarrow x = -2$, which is not possible for this situation. So the price of the cheapest book is $a_1 = 2$; the price of the middle book is $a_{16} = 2 + 15 \cdot 2 = 32$; and the price of the most expensive book is $a_{31} = 2 + 30 \cdot 2 = 62$, thus none of the statements are true.

32. E. If $a > b$, then $|a-b| = a-b$ and $|b-a| = -(b-a) = -b+a$. So $|a-b| + |b-a| = (a-b) + (-b+a) = 2a-2b$.

33. B Look for a pattern: 6:38 to 6:40 gives three changes; 6:40 to 6:50 gives 11 changes, 6:50 to 7:00 gives twelve changes. The total at this point would be 26. Thus we need 1 more change. Hence 7:00 to 7:01.

34. D. Fifteen have two legs, six have three legs, and two have five legs. Jacob's solution: We start out by setting x to the number with two legs, y to the number with 3 legs, and z to the number with 5 legs. This process gives us these equations:

$$\text{First day: } 2x + 3y + 5z = 58$$

$$\text{Second day: } 4x + 9y + 25z = 164 \quad \text{To solve this system of equations, we will}$$

$$\text{Third day: } 16x + 81y + 625z = 1976$$

use two pairs of equations to eliminate the x , then use the resulting pair of equations to eliminate the y and solve for z .

$$\left. \begin{array}{l} 2(2x + 3y + 5z) = 2 \cdot 58 \\ 4x + 9y + 25z = 164 \end{array} \right\} \Leftrightarrow \left. \begin{array}{l} 4x + 6y + 10z = 116 \\ 4x + 9y + 25z = 164 \end{array} \right\} \Rightarrow 3y + 15z = 48$$

$$\left. \begin{array}{l} 4(4x + 9y + 25z) = 4 \cdot 164 \\ 16x + 81y + 625z = 1976 \end{array} \right\} \Leftrightarrow \left. \begin{array}{l} 16x + 36y + 100z = 656 \\ 16x + 81y + 625z = 1976 \end{array} \right\} \Rightarrow 45y + 525z = 1320$$

So now we have the system

$$\left. \begin{array}{l} 3y + 15z = 48 \\ 45y + 525z = 1320 \end{array} \right\} \Leftrightarrow \left. \begin{array}{l} 15(3y + 15z) = 15 \cdot 48 \\ 45y + 525z = 1320 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 45y + 225z = 720 \\ 45y + 525z = 1320 \end{array} \right\} \Rightarrow 300z = 600$$

So $z = 2$, and substituting this into

$$45y + 525(2) = 1320 \Rightarrow 45y + 1050 = 1320 \Rightarrow 45y = 270 \Rightarrow y = 6. \text{ Now}$$

substituting the values for y and z into

$$2x + 3(6) + 5(2) = 58 \Leftrightarrow 2x + 18 + 10 = 58 \Rightarrow 2x = 30 \Rightarrow x = 15, \text{ so}$$

$$x + y + z = 15 + 6 + 2 = 23.$$

35. C. Let x be the number of employees. Then

$$\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 26 \Rightarrow 12 \left(\frac{x}{2} + \frac{x}{3} + \frac{x}{4} \right) = 12(26) \Rightarrow 6x + 4x + 3x = 312 \Rightarrow 13x = 312, \text{ so}$$

$$x = 24.$$

36. A.

$$\frac{4 + 20 + x}{3} = \frac{y + 16}{2} \Rightarrow 2(24 + x) = 3(y + 16) \Rightarrow 48 + 2x = 3y + 48 \Rightarrow 2x = 3y$$

$$\text{so the ratio } \frac{y}{x} = \frac{3}{2}.$$

37. A. There are 36 possible ways to roll two dice: (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), ... (6,4), (5,4), (6,6). Of these possibilities, there are 9 rolls for

which the larger of the two numbers gives a score of 3 or less. So the probability of scoring a 3 or less is $\frac{9}{36} = \frac{1}{4}$.

38. A. $45x = 1350 \Rightarrow x = 40$.

39. E. $2f(x) = 2f\left(2 \cdot \frac{x}{2}\right) = 2\left(\frac{2}{2 + \cancel{x/2}}\right) = 2\left(\frac{4}{4 + x}\right) = \frac{8}{4 + x}$.

40. D. Since m and n must both be positive, it follows that $n > 2$ and $m > 4$. Because $\frac{4}{m} + \frac{2}{n} = 1$ is equivalent to $(m-4)(n-2) = 8$, we need only find all the ways of writing 8 as a product of positive integers. The 4 ways are $1 \cdot 8, 2 \cdot 4, 4 \cdot 2, 8 \cdot 1$ corresponds to 4 solutions, $(m, n) = (5, 10), (6, 6), (8, 4)$, and $(12, 3)$.