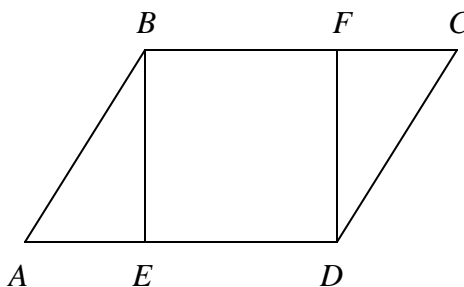


Geometry
State Mathematics Contest Finals
May 5, 2005

1. Given the conditional statement: "If two lines are parallel, then the lines do not intersect." Which of the following conditionals is equivalent to the given statement?
- a. "If two lines do not intersect, then the lines are parallel."
 - b. "If two lines intersect, then the lines are not parallel."
 - c. "If two lines are not parallel, then the lines intersect."
 - d. "If two lines are not parallel, then the lines do not intersect."
 - e. "If two lines do not intersect, then the lines are not parallel."

2. What is the area of an equilateral triangle inscribed in a circle of radius 4 *cm*?
- a. 12 cm^2 b. 16 cm^2 c. $12\sqrt{2} \text{ cm}^2$ d. $12\sqrt{3} \text{ cm}^2$ e. $16\sqrt{3} \text{ cm}^2$
3. In the diagram, *ABCD* is a parallelogram, and *BFDE* is a square. If *AB* = 20 *cm* and *CF* = 16 *cm*, what is the perimeter of the parallelogram *ABCD* ?

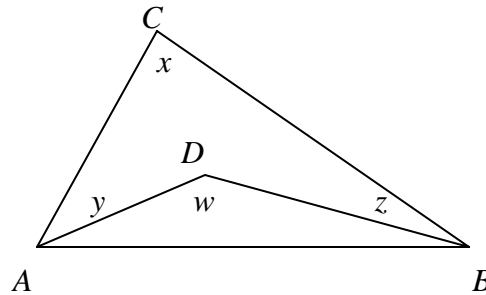
- a. 72 *cm*
- b. 78 *cm*
- c. 86 *cm*
- d. 92 *cm*
- e. 96 *cm*



4. A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?
- a. 20% b. 25% c. 36% d. 56.25% e. 64%
5. Three sides of a regular hexagon, no two of which share a vertex of the hexagon, are extended to form a triangle. The perimeter of the triangle thus formed is how many times the perimeter of the original hexagon?
- a. $\frac{3}{2}$ b. 2 c. $\frac{5}{2}$ d. 3 e. $2\sqrt{3}$

6. In $\triangle ABC$ shown, D is some interior point, and x, y, z, w are the measures of the indicated angles in degrees. Solve for x in terms of $y, z,$ and w .

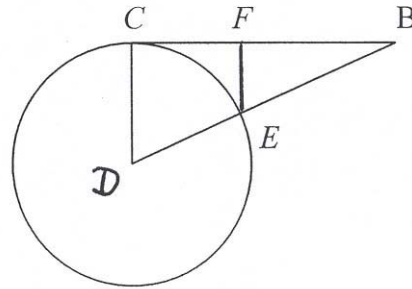
- a. $x = w - y - z$
 b. $x = w - 2y - 2z$
 c. $x = 2w - y - z$
 d. $x = 180^\circ - w + y + z$
 e. $x = 180^\circ + w - y - z$



7. Given three segments of length $x, 11 - x,$ and $x - 4,$ respectively. Which of the following indicates the set of all numbers x such that the three segments could be the lengths of the sides of a triangle?
- a. $x > 4$ b. $0 < x < 11$ c. $4 < x < 11$ d. $5 < x < 7$ e. $5 < x < 11$
8. Which of the following sets of triples could not be the lengths of the sides of an obtuse triangle?
- a. 2, 2, 3 b. 2, 3, 4 c. 3, 3, 5 d. 4, 5, 7 e. 5, 6, 7
9. A 25-foot ladder is placed against a vertical wall of a building. The foot of the ladder is 7 feet from the base of the building. If the top of the ladder slips 4 feet down the wall, how far will the foot of the ladder slide away from the wall?
- a. 4 feet b. 5 feet c. 8 feet d. 9 feet e. 15 feet
10. A rhombus is formed by two radii and two chords of a circle whose radius is 16 cm. Find the area of the rhombus.
- a. 128 cm^2 b. $128\sqrt{3} \text{ cm}^2$ c. 256 cm^2 d. $256\sqrt{3} \text{ cm}^2$ e. None of these
11. The line through points $(m, -9)$ and $(7, m)$ has slope m . What is the y-intercept of this line?
- a. -18 b. -6 c. 0 d. 6 e. 18
12. A small tree 5 feet from a lamp post casts a shadow 4 feet long. If the lamp post were 2 feet higher, the shadow would only be 3 feet long. How tall is the tree?
- a. $2\frac{2}{3}$ feet b. $3\frac{2}{5}$ feet c. $3\frac{1}{2}$ feet d. $4\frac{2}{3}$ feet e. $4\frac{4}{5}$ feet

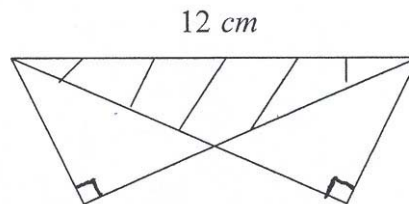
13. In the diagram, \overline{BC} is tangent to the circle centered at D . The diameter is 12 and $BC = 8$. Also, $\overline{FE} \parallel \overline{CD}$. Find FB .

- a. $2\frac{2}{5}$
 b. $3\frac{1}{5}$
 c. 4
 d. $4\frac{4}{5}$
 e. $5\frac{1}{3}$



14. How many regular polygons have integral interior angle measure?
 a. 20 b. 21 c. 22 d. 23 e. More than 23
15. Find the length-to-width ratio of a book if the rectangle formed by the closed book is similar to the rectangle formed by the open book.
 a. 2:1 b. $\sqrt{3}:1$ c. 3:2 d. $\sqrt{2}:1$ e. 4:3
16. Two congruent $30^\circ - 60^\circ - 90^\circ$ triangles are placed, as shown, so that they overlap partly and their hypotenuses coincide. If the hypotenuse is 12 cm, find the area common to both triangles.

- a. $6\sqrt{3} \text{ cm}^2$
 b. $8\sqrt{3} \text{ cm}^2$
 c. $9\sqrt{3} \text{ cm}^2$
 d. $12\sqrt{3} \text{ cm}^2$
 e. 24 cm^2

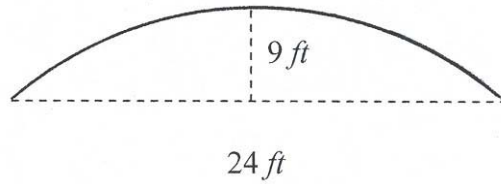


17. A triangle with integral sides has perimeter 8 cm. Find the area of the triangle.

- a. $2\sqrt{2} \text{ cm}^2$ b. $\frac{16}{9}\sqrt{3} \text{ cm}^2$ c. $2\sqrt{3} \text{ cm}^2$ d. 4 cm^2 e. $4\sqrt{2} \text{ cm}^2$

18. A circular arch having width 24 feet and height 9 feet is to be constructed. What is the radius of the circle of which the arch is an arc?

- a. 10 feet
- b. 12.5 feet
- c. 13.5 feet
- d. 14 feet
- e. 15 feet



19. Find the area of the quadrilateral $ABCD$ with vertices $A(-2, 0)$, $B(0, -4)$, $C(4, -2)$, and $D(2, 2)$.

- a. 12 sq. units
- b. 16 sq. units
- c. 20 sq. units
- d. 32 sq. units
- e. 40 sq. units

20. A “tire” in the shape of an equilateral triangle with side of length 6 cm is placed on a set of axes; the figures show the initial position (figure 1) and each position until the “tire” reaches its initial position again (figure 4).

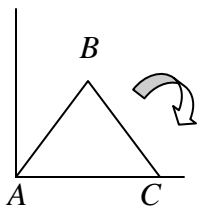


figure 1

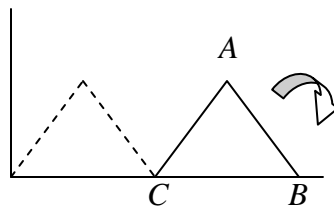


figure 2

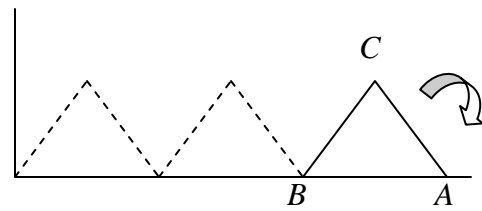


figure 3

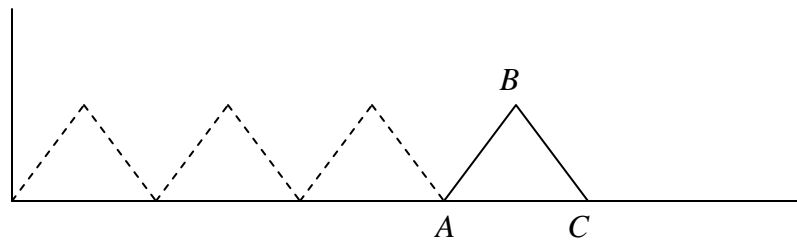


figure 4

How far has vertex A traveled as the “tire” goes from the alignment in figure 1 to the alignment in figure 4?

- a. 4π cm
- b. 18 cm
- c. 6π cm
- d. 24 cm
- e. 8π cm

21. In $\triangle ABC$, $m\angle ABC = 120^\circ$, $AB = 3$, and $BC = 4$. If perpendiculars constructed to \overline{AB} at A and to \overline{BC} at C meet at D , find CD .

- a. 3 b. $\frac{8\sqrt{3}}{3}$ c. 5 d. $\frac{11}{2}$ e. $\frac{10\sqrt{3}}{3}$

22. The sides of a regular polygon of n sides, $n > 4$, are extended to form a star. Find the number of degrees at each point of the star.

- a. $\frac{180^\circ}{n}$ b. $\frac{360^\circ}{n}$ c. $180^\circ - \frac{90^\circ}{n}$ d. $\frac{(n-4)180^\circ}{n}$ e. $\frac{(n-2)180^\circ}{n}$

23. How many noncongruent rectangles with integral sides have not more than 25 square units in their areas?

- a. 45 b. 46 c. 47 d. 49 e. 52

24. Suppose that a ray of light leaves the point $(3, 4)$, reflects off the y -axis towards the x -axis, reflects off the x -axis, and finally arrives at the point $(8, 2)$. Find the value of x .

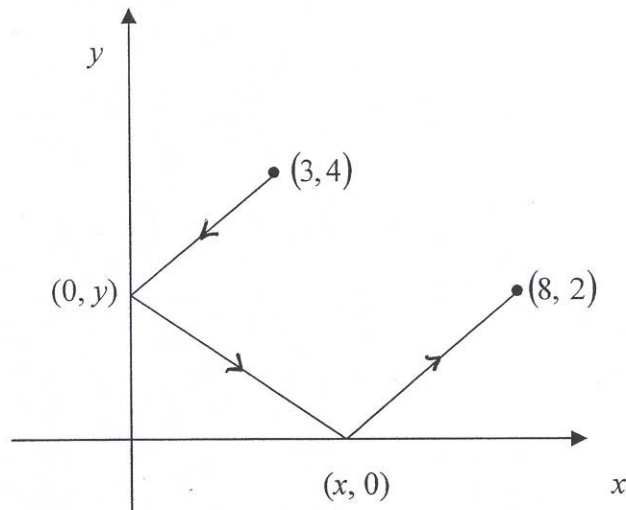
a. $x = 4\frac{1}{3}$

b. $x = 4\frac{1}{2}$

c. $x = 4\frac{2}{3}$

d. $x = 5$

e. $x = 5\frac{1}{3}$

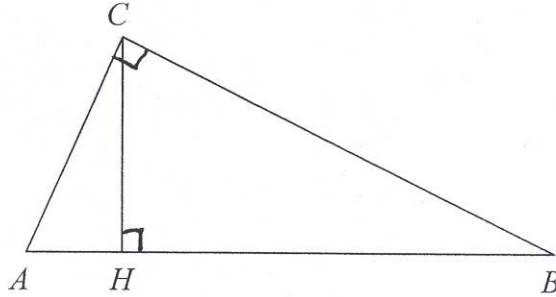


25. A square and an equilateral triangle have the same perimeter. Let A be the area of the circle circumscribed about the square and B be the area of the circle circumscribed about the triangle. Find $A:B$.

- a. 9:16 b. 3:4 c. 27:32 d. $3\sqrt{6}:8$ e. 1:1

26. $\triangle ABC$ is a right triangle with hypotenuse \overline{AB} and $AC = 15$ cm. Altitude \overline{CH} divides \overline{AB} into segments \overline{AH} and \overline{HB} , with $HB = 16$ cm. Find the area of $\triangle ABC$.

- a. 120 cm^2
 b. 144 cm^2
 c. 150 cm^2
 d. 216 cm^2
 e. None of these



27. A pentagon is formed by cutting a triangular corner from a rectangular piece of paper. The five sides of the pentagon have lengths 13, 19, 20, 25, and 31, although this is not necessarily their order around the pentagon. Find the area of the pentagon.

- a. 459 sq. units b. 600 sq. units c. 680 sq. units d. 720 sq. units e. 745 sq. units

28. Let $\triangle ABC$ have vertices $A(-4, -3)$, $B(6, -1)$, and $C(2, 5)$. Find the length of the median from C to \overline{AB} .

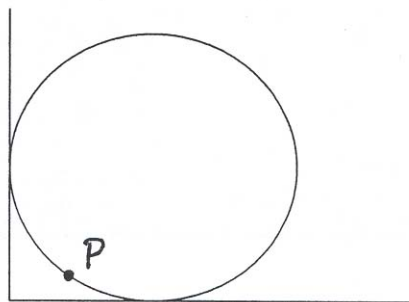
- a. $\sqrt{26}$ b. $\sqrt{50}$ c. $\sqrt{53}$ d. $\sqrt{89}$ e. $\sqrt{104}$

29. Twice the measure of the supplement of an angle is added to three times the measure of the complement of the same angle. The sum is the measure of an interior angle of a regular nine-sided polygon. What is the measure of the supplement of the angle?

- a. 82° b. 86° c. 90° d. 94° e. None of these

30. The circular table in the diagram is pushed against two perpendicular walls. The point P on the edge of the table is a distance 2 dm from one wall and a distance of 9 dm from the other wall as shown in the figure. What is the radius of the table?

- a. 11 dm
 b. $9\sqrt{2}$ dm
 c. $9\sqrt{3}$ dm
 d. 17 dm
 e. 18 dm

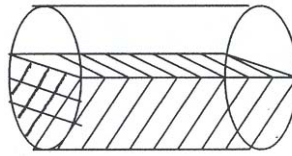


31. Let $\triangle XOY$ be a right triangle with $m\angle XOY = 90^\circ$. Let M and N be the midpoints of legs \overline{OX} and \overline{OY} , respectively. If $XN = 19$ and $YM = 22$, find XY .

- a. 26 b. $13\sqrt{5}$ c. 32.5 d. 41 e. None of these

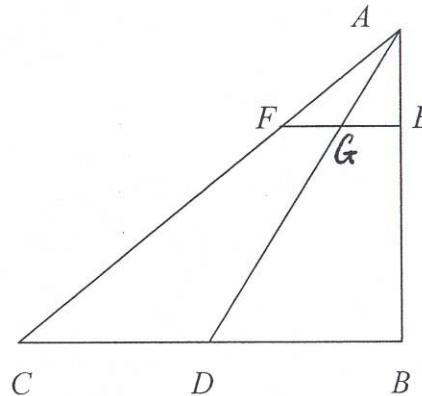
32. A spherical tank with height 22 inches and diameter 18 inches is lying on its side. Water is added to the tank to a depth of 13.5 inches. Use the fact that one gallon is approximately 231 cubic inches to determine the approximate number of gallons of water in the tank.

- a. 10.8 gal
 b. 12.8 gal
 c. 17.5 gal
 d. 16.1 gal
 e. 19.5 gal



33. In the diagram, $\angle ABC$ is a right angle. Point D is on \overline{BC} , and \overline{AD} bisects $\angle CAB$. Points E and F are on \overline{AB} and \overline{AC} , respectively, so that $AE = 3$ cm, $EB = 9$ cm, $AF = 5$ cm, and $FC = 15$ cm. Also, \overline{AD} intersects \overline{FE} at point G . Find the area of quadrilateral $DCFG$.

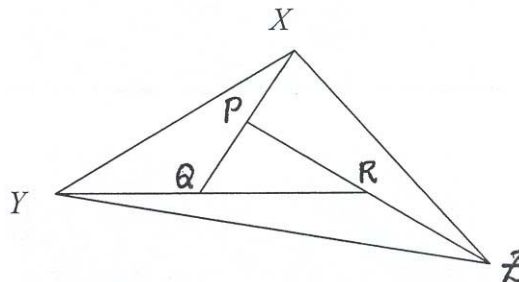
- a. 48 cm²
 b. $50\frac{5}{8}$ cm²
 c. $56\frac{1}{4}$ cm²
 d. 60 cm²



- e. None of these

34. $\triangle PQR$ has side \overline{QP} extended to X so that $QP = PX$, \overline{PR} extended to Z so that $PR = RZ$, and \overline{RQ} extended to Y so that $RQ = QY$. If the area of $\triangle XYZ$ is 3360 sq. units, find the area of $\triangle PQR$.

- a. 420 sq. units
 b. 480 sq. units
 c. 560 sq. units
 d. 672 sq. units
 e. 840 sq. units



35. A circle centered at A with radius of 1 and a circle centered at B with radius of 4 are externally tangent. A third circle is tangent to the first two circles and to one of their common external tangents as shown. Find the radius of the third circle.

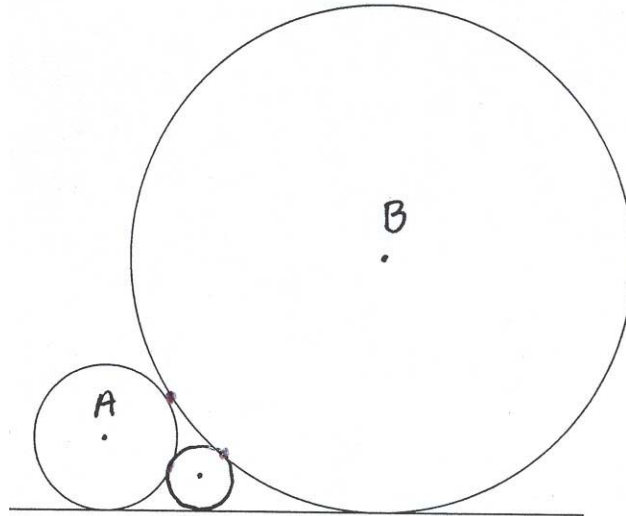
a. $\frac{1}{3}$

b. $\frac{2}{5}$

c. $\frac{5}{12}$

d. $\frac{4}{9}$

e. None of these



36. A 400-meter race is to be run on a track that is 400 meters long when measured along the innermost lane. The track consists of straight parallel sides, semicircular ends, and eight running lanes, each 1.23 meters wide. How far forward should the starting positions in neighboring lanes be marked so that the distance run by each competitor is the same?

a. $1.23 m$

b. πm

c. $2.46 m$

d. $1.23\pi m$

e. $2.46\pi m$

37. I have an unusual dog run in my yard. A 50-foot rope is tied at each end to two stakes that are 14 feet apart. My dog, Elvis, is tethered to the rope, but the tether is loose and slides freely along the rope between the stakes. I lay tree bark over the area of my yard that Elvis can reach. What is the area of the region that Elvis can reach?

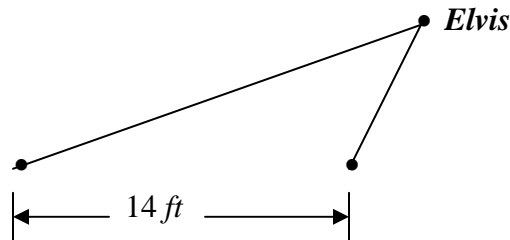
a. $576\pi \text{ ft}^2$

b. $600\pi \text{ ft}^2$

c. $625\pi \text{ ft}^2$

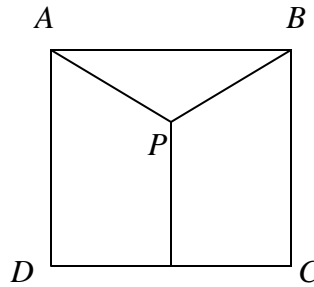
d. $700\pi \text{ ft}^2$

e. None of these



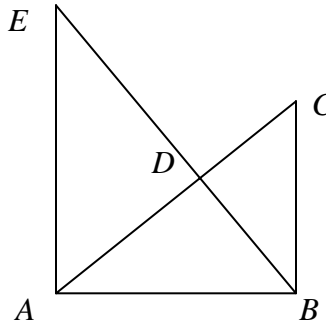
38. $ABCD$ is a square with $AB = s$. Point P is an interior point such that AP , BP , and the distance from P to \overline{CD} are all equal. Find this distance.

- a. $\frac{5}{8}s$ b. $\frac{2}{3}s$
 c. $\frac{\sqrt{2}}{2}s$ d. $\frac{3}{4}s$
 e. $\frac{\sqrt{3}}{2}s$



39. In the figure, $\angle EAB$ and $\angle ABC$ are right angles, $AB = 4$ cm, $BC = 6$ cm, $AE = 8$ cm, and \overline{AC} and \overline{BE} intersect at D . Find the area of $\triangle ABD$.

- a. $5\frac{1}{7} \text{ cm}^2$
 b. 6 cm^2
 c. $6\frac{6}{7} \text{ cm}^2$
 d. 8 cm^2
 e. $9\frac{1}{7} \text{ cm}^2$



40. Given trapezoid $ABCD$ with $\overline{AB} \parallel \overline{CD}$, $AB = a$, $CD = b$, and $\overline{MN} \parallel \overline{AB}$ such that trapezoid $ABNM$ has area equal to trapezoid $MNCD$. If $MN = x$, solve for x in terms of a and b .

- a. $x = \frac{a+b}{2}$ b. $x = \sqrt{ab}$
 c. $x = \frac{2ab}{a+b}$ d. $x = \sqrt{\frac{a^2 + b^2}{2}}$
 e. None of these

