

Algebra II
State Mathematics Contest Finals
May 4, 2006

1. The number of integral values of x that satisfy the inequality $|x + 10| > |2x + 1|$ is
- a. 5 b. 8 c. 11 d. 12 e. More than 12
2. $\frac{6}{3 - \sqrt{3 - x}}$ is equivalent to
- a. $\frac{36}{6 + x}$ b. $\frac{3 + \sqrt{3 - x}}{1 + x}$ c. $\frac{4\sqrt{3 - x}}{1 + x}$
- d. $\frac{18 + 6\sqrt{3 + x}}{x^2}$ e. None of these
3. Which of the following primes is not a factor of $55^{100} + 55^{101} + 55^{102}$?
- a. 3 b. 5 c. 7 d. 11 e. 13
4. The sum of two numbers is 6, and the sum of their reciprocals is $\frac{2}{5}$. What is the sum of their cubes?
- a. -54 b. -27 c. 27 d. 54 e. 162
5. If $y = \frac{x - 1}{1 - 2x}$ and $x \neq \frac{1}{2}$, then $\frac{y - 1}{1 - 2y} =$
- a. x b. $\frac{3x - 2}{3 - 4x}$ c. $\frac{2 - 3x}{3 + 4x}$ d. $\frac{3x + 2}{3 + 4x}$ e. $\frac{3x + 2}{3 - 4x}$
6. If A, B, C are numbers for which $\frac{x^2 - 10x - 36}{x(x - 3)^2} = \frac{A}{x} + \frac{B}{x - 3} + \frac{C}{(x - 3)^2}$, then $A + B + C =$
- a. -36 b. -18 c. 3 d. 8 e. 10

7. How many positive integers n have the property that 62 is divisible by $n - 4$?
- a. 0 b. 2 c. 4 d. 6 e. None of these
8. The sum of the first five terms of an arithmetic sequence is 40, and the sum of the first ten terms of the sequence is 155. What is the sum of the first fifteen terms of the sequence?
- a. 195 b. 230 c. 345 d. 390 e. 780
9. Arnold, Barry, and Carl together received \$270. If Barry gave half of his money to Carl, then Carl would have twice as much as Arnold. However, if Barry gave a third of his money to Arnold, then Carl and Arnold would have the same amount. How much money does Arnold have?
- a. \$75 b. \$90 c. \$105 d. \$144 e. None of these
10. If $\log_8 3 = p$ and $\log_3 5 = q$, then find $\log_{10} 5$ in terms of p and q .
- a. $1 - 3pq$ b. $q + 3p$ c. $\frac{2pq}{p+q}$ d. $\frac{3pq}{p+q}$ e. $\frac{3pq}{1+3pq}$
11. If $f^{-1}\left(\frac{2}{x}\right) = x + 2$, then $f(2x) =$
- a. $\frac{1}{x-1}$ b. $\frac{1}{x+1}$ c. $x - 1$ d. $\frac{x}{x+1}$ e. $\frac{x-1}{x}$
12. Factor the expression: $(x^2 + 6)^{7/2} + 2x(x^2 + 6)^{5/2} + x^2(x^2 + 6)^{3/2}$
- a. $(x^2 + 6)^{3/2}(x+2)^2(x+3)^2$ b. $(x^2 + 6)^{3/2}(x^2 + x + 6)^2$
- c. $(x^2 + 6)^{3/2}(x-2)^2(x+3)^2$ d. $(x^2 + 6)^{1/2}(x^2 + x + 6)^2$
- e. $(x^2 + 6)^{1/2}(x+2)^2(x+3)^2$

13. The fraction $\frac{36}{79} = \frac{1}{a + \frac{1}{b + \frac{1}{c}}}$ where a , b , and c are natural numbers.

Find the numerical value of $a + b + c$.

- a. 7 b. 11 c. 13 d. 14 e. 17

14. The tax rates for a small business are 30% federal and 10% state. If a tax is not paid on a tax (i.e. federal tax is subtracted before calculating state tax), what is the total tax for a business whose taxable income is \$97,000?

- a. \$33,271 b. \$33,853 c. \$35,890 d. \$38,800 e. None of these

15. Which of the following equations have the same graph?

I. $y = x - 2$ II. $y = \frac{x^2 - 4}{x + 2}$ III. $(x + 2)y = x^2 - 4$

- a. I and II only b. I and III only c. II and III only
d. I, II, and III e. None. All the equations have different graphs

16. Let $x > 0$. The terms of the sequence $\sqrt[3]{x}$, \sqrt{x} , $\sqrt[3]{x^2}$, ... are in geometric progression. What will be the fourth term?

- a. $\sqrt[4]{x^3}$ b. $\sqrt[5]{x^4}$ c. $\sqrt[6]{x^5}$ d. x e. $\sqrt{x^3}$

17. The maximum value of $f(x) = -x^2 + 2bx + c$ is

- a. b b. c c. $c - b^2$ d. $c + b^2$ e. $c + 3b^2$

18. The product of the solutions to $(\ln x)^3 = \ln(x^3)$ is

- a. 0 b. 1 c. e^{-3} d. e e. $e^{\sqrt{x}}$

19. Let $f(x) = \frac{x^3 - 2}{x^3 + 1}$. If $(f \circ g)(x) = x$, then $g(x) =$

a. $(x^3 + 1)^3 - 2$

b. $\frac{x^3 + 1}{x^3 - 2}$

c. $\sqrt[3]{\frac{2-x}{x-1}}$

d. $\sqrt[3]{\frac{2x+1}{x-1}}$

e. $\sqrt[3]{\frac{x+2}{1-x}}$

20. A function $f(x)$ is defined by the relationship $\frac{10}{f(x-3)} + \frac{3}{f(10-x)} = \frac{130}{x^2}$.

Find $f(7)$.

a. -3

b. $-\frac{1}{3}$

c. $\frac{20}{313}$

d. $\frac{90}{139}$

e. None of these

21. Let $P(x)$ be a polynomial such that when $P(x)$ is divided by $(x - 19)$, the remainder is 99; and when $P(x)$ is divided by $(x - 99)$, the remainder is 19. What is the remainder when $P(x)$ is divided by $(x - 19)(x - 99)$?

a. $-x + 80$

b. $-x + 118$

c. 0

d. $x + 80$

e. $x + 118$

22. A parabolic function with vertex $(3, 19)$ and y-intercept = 1 passes through which of the following points?

a. $(5, 11)$

b. $(5, 13)$

c. $(5, 15)$

d. $(5, 18)$

e. $(5, 25)$

23. The first four terms in an arithmetic sequence are $x + y$, $x - y$, xy , $\frac{x}{y}$ in that order.

What is the fifth term?

a. $-\frac{15}{8}$

b. $-\frac{6}{5}$

c. 0

d. $\frac{27}{20}$

e. $\frac{123}{40}$

24. Solve the equation, $2^{-4x} + 0.25 \cdot a^2 = a \cdot 2^{-2x}$, for x :

a. $x = \frac{\log_2 a - 1}{2}$

b. $x = 2 \log_2 a - 1$

c. $x = \frac{1 \pm \sqrt{1 - \log_2 a}}{2}$

d. $x = 2 \log_2(a - 1)$

e. $x = \frac{1 - \log_2 a}{2}$

25. For each real number x , let $f(x)$ be the minimum of the numbers $3x + 1$, $x + 1$, $-2x + 5$. Find the maximum value of $f(x)$.

a. $\frac{4}{5}$

b. 1

c. $1\frac{1}{3}$

d. $2\frac{1}{3}$

e. $3\frac{2}{5}$

26. Simplify $\frac{(\sqrt{3} + i)^3}{i + \frac{1+i}{1-i}}$ where $i = \sqrt{-1}$.

a. 4

b. $\sqrt{3} - 3i$

c. $\frac{3\sqrt{3} - 3i}{2}$

d. $4 - 4i$

e. None of these

27. Find the sum of the first 2006 terms of the following infinite series:

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} + \dots$$

a. $\frac{1}{32192286}$

b. $\frac{1}{16096143}$

c. $\frac{2006}{16096143}$

d. $\frac{2006}{4013}$

e. $\frac{2006}{4011}$

28. On December 1, Mr. Gates decides to buy a hot stock from a commission-free brokerage house. His purchase price of Volatile Inc is \$1000, and he is pleased to see the stock soar 20% the same day he buys it. The next day, however, Volatile stock plunges 20%. On December 3, the stock is back up 20%; but on December 4, it's back down 20%. Volatile's stock continues its yo-yo trip throughout the month, rising 20% on the odd days and falling 20% on the even days. On December 31, after the 20% gain, Mr. Gates sells his stock. How much will he receive?

- a. \$352.63 b. \$542.09 c. \$650.50 d. \$960 e. \$1200

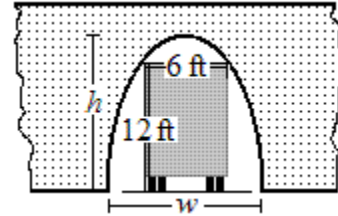
29. If the solutions (for x) of $x^2 + px + q = 0$ are the cubes of the solutions (for x) of $x^2 + mx + n = 0$, express p in terms of m and n .

- a. $p = n^3 - 3mn$ b. $p = m^3 - 3mn$ c. $p = m^3$
 d. $p = m^3 + n^3$ e. $p = m^3 + 3mn$

30. The equation $y = x^2 + 2ax + a$ represents a family of parabolas for all real values of a . Each of these parabolas passes through a common point. If this common point is (s, t) , find the numerical value of $s + t$.

- a. -1 b. $-\frac{1}{2}$ c. $-\frac{1}{4}$ d. 0 e. $\frac{1}{4}$

31. A divided highway into the city passes under a number of bridges. The arch over each lane is in the form of a semi-ellipse, where the height is equal to the width. What is the lowest bridge under which a truck 6 feet wide and 12 feet high can theoretically pass?

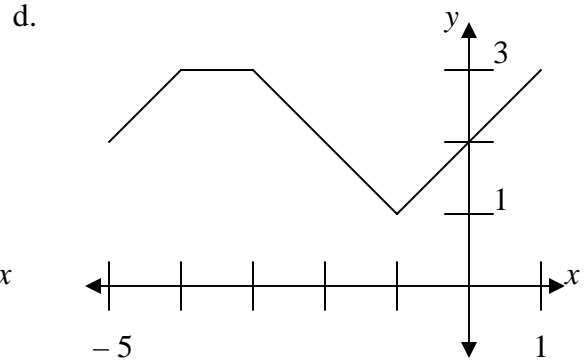
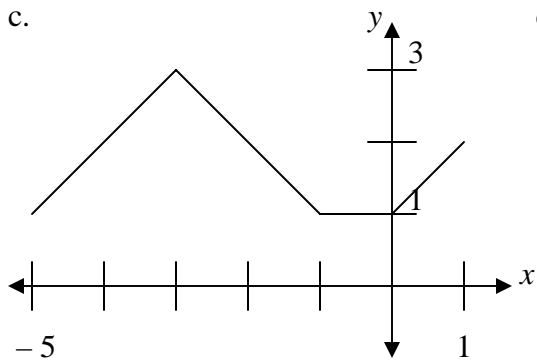
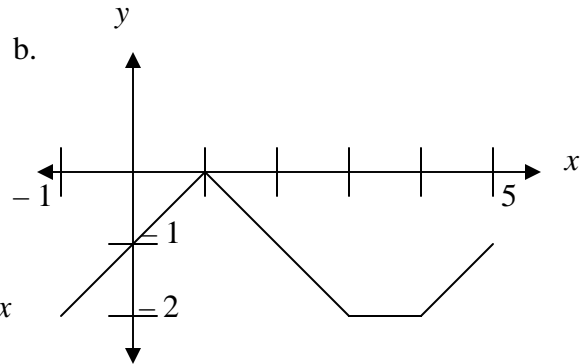
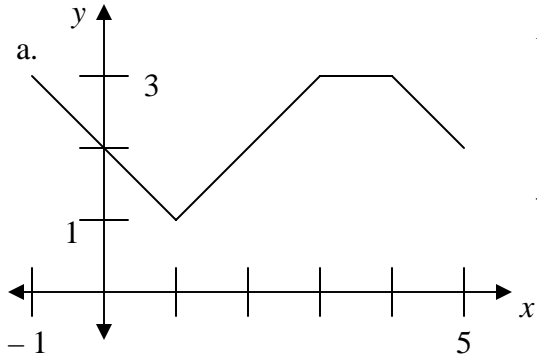
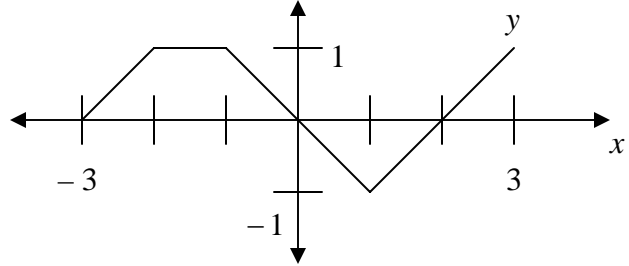


- a. $3\sqrt{5}$ feet b. $\sqrt{153}$ feet c. $6\sqrt{5}$ feet d. $2\sqrt{153}$ feet e. None of these

32. $\log_2 x + \log_4 x + \log_8 x =$

- a. $\log_{14} x$ b. $\frac{\log\left(\frac{11}{6}x\right)}{\log 2}$ c. $\frac{\log(x^6)}{\log 2048}$ d. $\frac{\log(x^{11})}{\log 64}$ e. None of these

33. If the graph at right represents the function $y = f(x)$ defined on the interval $[-3, 3]$, which of the following could represent the graph of $y = 2 - f(2 - x)$?



e. None of these

34. Find $i + 2i^2 + 3i^3 + 4i^4 + \dots + 2006i^{2006}$ where $i = \sqrt{-1}$.

a. $-1004 - 1003i$ b. $-1004 + 1003i$ c. 0 d. $1004 - 1003i$ e. $1004 + 1003i$

35. For any nonzero a and b , define a sequence as follows:

$$x_1 = a, x_2 = b, x_{n+2} = \frac{x_{n+1} + 1}{x_n} \text{ for } n = 1, 2, 3, \dots$$

Find x_6 .

a. a b. b c. $\frac{a+1}{b}$ d. $\frac{a+b+1}{ab}$ e. $\frac{b+1}{a}$

36. A circle of radius 4 is centered at the origin; every second, its radius increases by 3 units. A second circle of radius 12 is centered at $(30, 0)$; every second, its radius decreases by 1 unit. This process continues until the circles meet. At that time, the point $(27, 4)$ lies
- a. inside the first circle. b. on the first circle. c. inside the second circle.
d. on the second circle. e. between the circles.
37. Find the coefficient of x^3 in the expansion of $(1 + x + x^2)^{12}$.
- a. 132 b. 220 c. 352 d. 552 e. 2024
38. Exactly two of the divisors of $4^{12} - 1$ are between 60 and 80. Find the sum of these two divisors.
- a. 126 b. 128 c. 129 d. 140 e. 142
39. Find the locus of points whose distance from the point $(-1, 0)$ is twice the distance from the point $(2, 3)$. What is an equation of this locus of points?
- a. $y = -x + 3$ b. $y = x^2 - 2x + 3$ c. $x^2 + y^2 - 6x - 8y + 17 = 0$
d. $x^2 + 4y^2 - 6x - 32y + 53 = 0$ e. None of these
40. Let the function $A(x, y)$ be defined by the following rule:
 $A(0, n) = n + 1,$
 $A(m, 0) = A(m - 1, 1),$
 $A(m, n) = A(m - 1, A(m, n - 1))$ where m and n are positive integers.
Find the numerical value of $A(2, 3)$.
- a. 5 b. 6 c. 7 d. 8 e. 9

Answers
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State Mathematics Contest Finals
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1. d	11. a	21. b	31. c
2. e	12. b	22. a	32. d
3. c	13. d	23. e	33. e
4. a	14. c	24. e	34. b
5. b	15. e	25. d	35. a
6. b	16. c	26. a	36. d
7. d	17. d	27. d	37. c
8. c	18. b	28. c	38. b
9. a	19. e	29. b	39. c
10. e	20. a	30. c	40. e

Tie Breakers:

Best of Three: 33, 37, 40

Sudden Death: 39, 38, 31, 29, 21, 19, 15