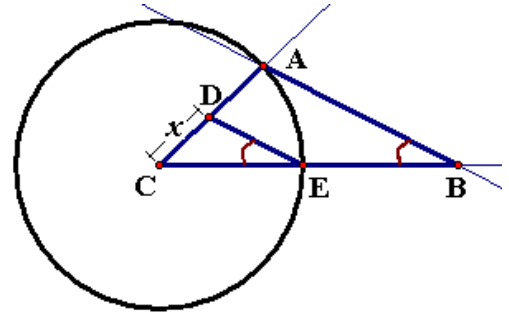


State Mathematics Finals: Geometry
May 3, 2007

1. What is the surface area of a rectangular solid with 5 cm height, 3 cm width, and 7 cm length?
- a. 71 cm^2 b. 105 cm^2 c. 140 cm^2 d. 142 cm^2 e. 166 cm^2

2. $\triangle ABC$ has a circle centered on vertex C that passes through points A and E. If $\angle ABC \approx \angle DEC$, $AC = 1$ and $CD = x$, what is the distance between E and B?

- a. $2x$ b. $\sqrt{1-x^2}$ c. $1+x$
d. $\frac{1}{x}$ e. $\frac{1-x}{x}$



3. A quadrilateral has these properties: 3 of its angles are congruent to one another, no set of three vertices are co-linear, and its largest angle is three times as large as its smallest one. What is the measure of the smallest angle?

- a. 36° b. 60° c. 72° d. 90° e. 108°

4. How many different seven digit numbers can you form with the digits 1, 2, 3, 3, 5, 5, 5?

- a. 420 b. 840 c. 1260 d. 5040 e. 16,384

5. If May 3, 2007 is a Thursday, what day of the week is May 3, 2017?

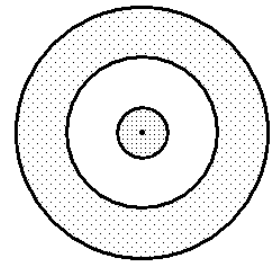
- a. Monday b. Tuesday c. Wednesday d. Friday e. none of these

6. What is the length of the hypotenuse of a right triangle that has an area of 10 and its shortest side has length a ?

- a. $\frac{20}{a}$ b. $\frac{\sqrt{25+a^2}}{a}$ c. $\frac{\sqrt{400+a^2}}{a}$ d. $\frac{\sqrt{400+a^4}}{a}$ e. $\sqrt{100+a^2}$

7. Three concentric circles with diameters 1, 3 and 5 form a target with 3 regions. How many times larger is the outer ring than the inner circle?

- a. 5 b. 9 c. 15 d. 25 e. none of these



8. A cube that is inside a sphere has its eight corners touching the sphere. What is the ratio of the volume of the sphere to the volume of the cube?

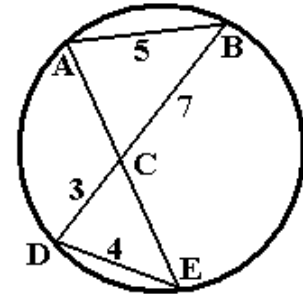
- a. $\frac{1}{2} \pi$ b. $\frac{\sqrt{3}}{4} \pi$ c. $\frac{\sqrt{3}}{2} \pi$ d. $\frac{\sqrt{2}}{2} \pi$ e. $\frac{3}{2} \pi$

9. Only one of the following statements is false, all the others are true. Find the false statement.

- a. Statement “b”, “c”, or “d” is false.
- b. If statement “c” is false then statement “d” is true.
- c. It is not true that statement “e” is false.
- d. Statements “b” and “c” have the same truth value.
- e. If statement “a” is true then “c” is false.

10. Given a circle with four chords, \overline{AB} , \overline{BD} , \overline{DE} and \overline{AE} , two of which intersect at C as shown, if $AB=5$, $BC=7$, $CD=3$ and $DE=4$, then what is the length of \overline{AE} ?

- a. 5.6 b. 9.35 c. 10
d. 11.85 e. none of these



11. A rectangular solid is created by stacking unit cubes (cubes with edges of length 1). This solid has a surface area of 52 square units and height of 2 units. Which possible value could its volume (in cubic units) have?

- a. 18 b. 22 c. 24 d. 26 e. 32

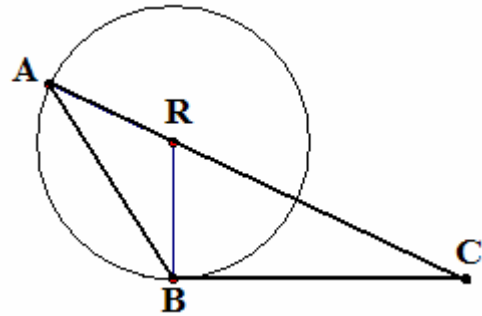
12. The length of a rectangle is decreased by 10 cm while the width is increased by 6 cm. If the result is a square whose area is equal to that of the original rectangle, how big is that area?

- a. 60 cm^2 b. 64 m^2 c. 225 cm^2 d. 256 cm^2 e. none of these

13. Given a triangle with sides of lengths: $\sqrt{3}$, $\sqrt{3}$, and 3, find the measure of its smallest angle.
- a. 0.52° b. 30° c. 45° d. 60° e. none of these
14. Given the two-digit numbers, 10 through 99, and the function, P , that multiplies the two digits together. If you repeatedly apply function P until you get a single digit number, what percentage of the time will that single digit number be 0?
- a. 10% b. $14\frac{4}{9}\%$ c. $18\frac{8}{9}\%$ d. 24% e. $26\frac{2}{3}\%$
15. What is the area of a right triangle with a 56 cm perimeter and a hypotenuse of length 25 cm?
- a. 48 cm^2 b. 84 cm^2 c. 168 cm^2 d. 700 cm^2 e. none of these
16. How many faces does a polyhedral solid with 18 vertices and 32 edges have?
- a. 14 b. 16 c. 18 d. 28 e. 48
17. This is the information about students participating in a field trip:
36 were female students,
13 were male seniors,
41 were not drama majors,
exactly half of the drama majors were male,
a third of the female students were drama majors but not seniors,
there were 15 senior drama majors 20% of which were male.
How many students were on the trip?
- a. 89 b. 90 c. 99 d. 105 e. none of these
18. How long is the hypotenuse of a right triangle if the longer leg has length $\sqrt{5}$ and the smallest angle is 36° ?
- a. 5 b. $5 - \sqrt{5}$ c. $0.5 + \sqrt{5}$ d. $\frac{11}{4}$ e. 2.7

19. A 14 inch diameter pizza is cut into 12 slices. What is the distance around one slice?
- a. $14 + \frac{1}{12}\pi$ b. $14 + 7\pi$ c. $14 + \frac{7}{3}\pi$ d. $14 + \frac{7}{6}\pi$ e. $28 + \frac{7}{3}\pi$
20. An orange with a 1.2 cm thick skin is half of its original volume after it is peeled. Approximately how big was the orange's original diameter?
- a. 4.80 cm b. 5.82 cm c. 8.19 cm d. 9.23 cm e. 11.63 cm
21. A rectangular solid with a black surface area and dimensions $10 \times 12 \times 4$ is cut into unit cubes. Assuming the solid's interior is not black, what fraction of these cubes has only one black side?
- a. $\frac{29}{120}$ b. $\frac{29}{60}$ c. $\frac{29}{48}$ d. $\frac{2}{3}$ e. $\frac{13}{15}$

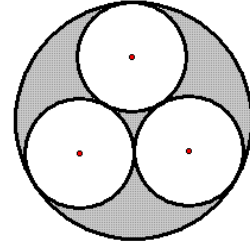
22. The point R lying on one of the edges of triangle $\triangle ABC$ is equidistant from vertices A and B. (See figure on right.) If $\angle ACB$ measures 28° , determine the measure of $\angle BAC$.



- a. 28° b. 54° c. 62°
d. 108° e. none of these

23. Find the remainder when 17^{2007} is divided by 5.
- a. 0 b. 1 c. 2 d. 3 e. 4
24. Find the height of a pyramid with a square base whose area is 16 cm^2 and whose lateral edges have length 5 cm.
- a. 3 cm b. $\sqrt{13}$ cm c. $\sqrt{25 - \frac{64}{3\sqrt{3}}}$ cm d. $5 - \frac{8}{\sqrt{3}}$ cm e. $\sqrt{17}$ cm

25. Three mutually tangent unit circles are circumscribed by a fourth circle as shown in the figure on the right. What proportion of the largest circle's area is shaded?



- a. $36\sqrt{3} - 62$ b. 0.25 c. $\frac{3}{2}\sqrt{3} - 2$
d. $6\sqrt{3} - 10$ e. $\frac{2-\sqrt{3}}{6}$
26. A ceiling fan with 45 cm long blades spins 132 rotations a minute. If a fly is sitting at the tip of the blade, approximately how fast is it traveling?
a. 18.4 km/hr b. 22.4 km/hr c. 37.3 km/hr d. 128.5 km/hr e. 373.2 km/hr
27. The interior angles of a pentagon have measures x , $x + 20^\circ$, $x + 40^\circ$, $x + 60^\circ$ and $x + 80^\circ$. What is the measure of the largest angle?
a. 68° b. 148° c. 184° d. 220° e. none of these
28. The area of a right triangle is 36. What is the shortest possible length of its hypotenuse?
a. $\sqrt{18}$ b. 6 c. $\sqrt{72}$ d. 12 e. none of these
29. Let k be a natural number and $n = 2k^2 + 1$. Which of the following statements are true for all values of n ?
i. n is NOT divisible by 2 ii. nk is divisible by 3 iii. n is NOT divisible by 5
a. i only b. i and ii c. i and iii d. none of them e. all of them
30. A rectangle is partitioned into four smaller rectangular pieces, three of which have areas, 15, 21 and 32. The fourth one is not given. What is the area of the entire rectangle?

15	21
32	

- a. 45 b. 68 c. 86.2
d. 112.8 e. none of these

31. The ratio of the dimensions of a rectangular solid is $5 : 4 : 3$, and the ratio of this solid's volume to its surface area is $2 : 1$. What is the length of the solid's shortest side?

- a. 3 b. $3\sqrt{13}$ c. 9.4 d. $12\sqrt{53}$ e. none of these

32. Six performers, 4 men and 2 women, walk on the stage to play Shakespeare's *Romeo and Juliet*. The roles of Romeo and Juliet are assigned to the performer by having the audience pull names out of a hat. Assuming this process is random what is the probability that Romeo will be played by a male and Juliet by a female?

- a. $\frac{4}{15}$ b. $\frac{8}{15}$ c. $\frac{2}{9}$ d. $\frac{1}{2}$ e. $\frac{1}{4}$

33. $2k + 1$ evenly spaced points on a circle are numbered 0 thru $2k$ in succession. Determine the measure of the angle formed at point k by connecting the point 0 to point k to point $2k$?

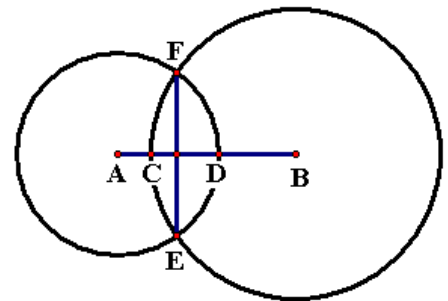
- a. $\frac{180^\circ}{k}$ b. $\frac{360^\circ}{k}$ c. $\frac{180^\circ}{2k+1}$ d. $\frac{360^\circ}{2k+1}$ e. none of these

34. Let x be the largest possible product of a 3-digit and a 4-digit number. These two numbers are formed using each of the digits 1 – 7 exactly once. Which statement concerning x is true?

- a. $x < 4,835,000$ b. $4,844,000 \leq x < 4,845,000$ c. $4,845,000 \leq x < 4,846,000$
 d. $4,846,000 \leq x < 4,847,000$ e. $4,847,000 \leq x$

35. Two circles centered at points A and B, respectively, intersect at points E and F as shown. These circles intersect segment \overline{AB} at points C and D. If $AC=1$ and $CD = DB = 2$, determine EF.

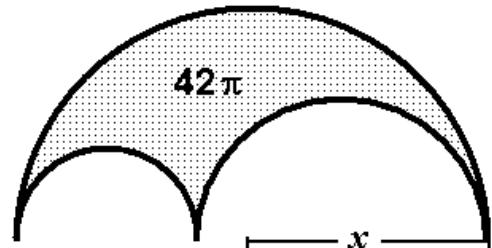
- a. 2.4 b. 3 c. $2\sqrt{5}$ d. $2\sqrt{7}$ e. 4.8



36. Find the area of a circle that circumscribes a triangle whose sides have lengths: 20, 21 & 29.
- a. $\frac{841}{4}\pi$ b. 420π c. 625π d. 145π e. $\frac{4900}{9}\pi$

37. How many distinct factors does the number 210 have?
- a. 4 b. 6 c. 12 d. 16 e. 24

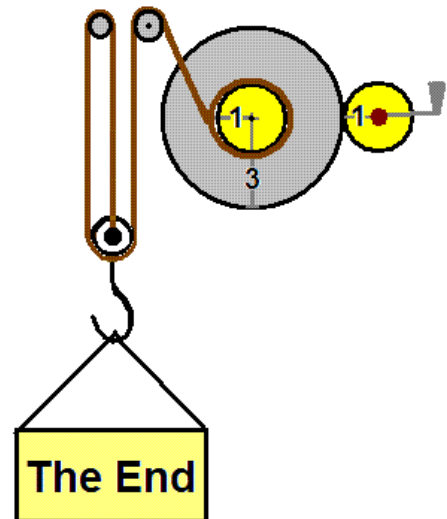
38. An *arbelos* is the region bordered by three mutually tangent circles with co-linear centers. See shaded region in the figure. If the area of this region is 42π and the radii of the two smaller circles have a ratio of 2 : 3, what is the radius, x , of the upper (and largest) circle?



- a. $2\sqrt{21}$ b. $5\sqrt{7}$ c. $7\sqrt{6}$ d. 35 e. none of these

39. What is the area of a quadrilateral that has three of its vertices spaced evenly around a circle of radius 6 while that circle's center is the fourth vertex?
- a. $18\sqrt{3}$ b. $9\sqrt{6}$ c. 36 d. $27\sqrt{3}$ e. $36\sqrt{3}$

40. A pulley constructed as shown in the figure on the right is designed to lift a sign. If one end of the rope is fixed to the pulley itself, while the other is on a spool with a 1 ft radius, approximately how much and in what direction must you turn the crank to lower the sign 3 feet?



- a. 0.48 counter clockwise turns
 b. 1.43 clockwise turns
 c. 1.43 counter clockwise turns
 d. 3.14 clockwise turns
 e. 4.30 counter clockwise turns