1. Answer d): We want \(a^2/4 = a\) so \(a = 4\).

2. Answer c): The areas of \(ABE\) and \(AFD\) each equal 1/4 of the area of \(ABCD\), and the area of \(ECF\) is 1/8 of the area of \(ABCD\). \(1 - 1/2 - 1/8 = 3/8\).

3. Answer b): The first marble falls a foot after 1/4 seconds. If we let \(y = 0\) at the bottom of the cliff, 
\[
y_2(t) = 64 - 16(t - 1/4)^2
\]
and \(y_2(2) = 15\)

4. Answer a): (I) is true with side lengths \(\{a, b, b\}\) if \(a < b\). (II) is false when \(2a < b\). (III) is false since \(\frac{b\sqrt{a^2 - (b/2)^2}}{2} = \frac{a\sqrt{b^2 - (a/2)^2}}{2} \Rightarrow a = b\).

5. Answer e): Thinking about the intersection of the oil with a disk of radius 6 perpendicular to the axis of the cylinder, we see that the volume is 12 \(\pi (\frac{6.5}{2})^2 = 12 \cdot 62^{2/3} = 265.33\)

6. Answer b): Let \(x = b'\) and consider \(\log_b(b') = \log_{b'}(b)\) or \(t = 1/t\). Then, \(t = \pm 1\) and \(x = b\) or \(1/b\).

7. Answer a): Think about placing ten 5 by 4 \(\pi\) rectangles side by side with the 5 inch edges adjacent. The path of the ant forms a straight line from the lower left hand corner of the leftmost rectangle to the upper right corner of the rightmost rectangle.

8. Answer b): The function \(x^{1/2}\) increases up to its maximum at \(x = e\) and decreases thereafter. Also, \(a^b = b^a \Rightarrow a^{3/4} = b^{3/4}\). Thus, if equation has a solution in integers, \(a < b, a = 1\) or 2. We see that only \(a = 2\) and \(b = 4\) solve the given equation.

9. Answer c): \(x + (1 - y) > (y - x), x + (y - x) > 1 - y, (1 - y) + (y - x) > x, 0 < x < 1/3, 2/3 < y < 1\) must all hold. Thus we need \((x, y)\) in the square \(0 < x < 1/3, 2/3 < y < 1\) which satisfy \(y < 1/2 + x\). These points lie in a right triangle with base and height 1/6, and \((1/72)/(1/9) = 1/8\).

10. Answer d): \(p + n + d + q = 101, p + 5n + 10d + 25q = 582, p + n = 15q, p = d + q + 6\) give \(p = 107 - 15q, n = 30q - 107, d = 101 - 16q\) so that \(q = 4, 5\) or 6.

11. Answer b): The side lengths of the triangle are the sums of the pairs of the radii. Thus the semiperimeter of the triangle is 144 + 225 + 256 = 625. Applying Heron’s Formula, the area is the square root of \(625 \cdot 144 \cdot 225 \cdot 256\).

12. Answer a): Applying Ptolemy’s Theorem to the cyclic quadrilateral \(APBC\), \(z|AB| = x|BC| + y|AC|\).

13. Answer d): \(|2iz + 4| = 2|z - 2i|\) so \(z = 3 + 2i\) gives the maximum.

14. Answer b): A given team wins both its games and is thus the unique winner of the tournament with probability 1/4. If no team wins both its games, the tournament ends in a three way tie.

OVER FOR SOLUTIONS TO THE REMAINING PROBLEMS.
16. Answer d): Edges 2, 4 and 6 satisfy the given conditions. If there were an edge of length 7, the sum of the lengths of the other two edges would be 5 and the product of their lengths would be 48/7. However, if \( x + y = 5, xy \) is at most 25/4.

17. Answer a): With \( x = \sqrt{b \cos(t)}, y = \sqrt{a \sin(t)} \), \( xy = \left(\sqrt{ab \sin(2t)}\right)/2 \).

18. Answer d): Let \( P = ABCDE \), in clockwise order. Suppose that \( AD \) and \( BE \) meet at \( Q \) and \( AC \) and \( BE \) meet at \( R \). Since \( \angle DAC = \pi/5 \), \( \Delta ABQ \) is isosceles with \( |BQ| = 1 \). Let \( s = |QR| \) and \( r = |RB| \); we want \( s \). The Law of Sines gives \( s/\sin(\pi/5) = r/\sin(2\pi/5) \Rightarrow r = \left(\frac{1 + \sqrt{5}}{2}\right)s \Rightarrow s = \frac{3 - \sqrt{5}}{2} \).

19. Answer e): \( \frac{1}{a_k \sqrt{a_{k+1}}} + a_{k+1} \sqrt{a_k} = \frac{1}{2} \left( \frac{1}{\sqrt{a_k}} - \frac{1}{\sqrt{a_{k+1}}} \right) \) so \( S(n) = \frac{1}{2} - \frac{1}{2\sqrt{2n+1}} \), and this last equals 1003/2007 if \( n = 2,014,024 \).

20. Answer e): Let \( n = 2\cdot5^m \cdot q \) where \( \gcd(q,10) = 1 \). Choose \( r \) so that \( 10^r \equiv 1 \pmod{q} \), and let \( x = 10^r \).

Then \( X = x^0 + x^1 + L + x^{r-1} \equiv 0 \pmod{q} \). Then \( 10^{\max(k,n)}X \) is a multiple of \( n \) of the desired form.

**Integer Answer Problems**

1. Answer: 91 We can assume \( a + 35 = b \) and \( ab = 7 \cdot 252 \). It follows that \( a = 28 \) and \( b = 63 \).

2. Answer: 36,000 \( P(6,4)P(5,3) + P(6,3)P(5,4) = 36,000 \)

3. Answer: 8 If \( n \) orange jellybeans are added, we want \( 1 - C(8,3)/C(8 + n,3) \geq 9/10 \).

4. Answer: 123 \( \sum_{k=0}^{n} (n + k)(2n - k) = 2n^2(n + 1) + n^2(n + 1)/2 - n(n + 1)(2n + 1)/6 = n(n + 1)(13n - 1)/6 \), and this exceeds the square of 2007 if \( n > 122 \).

5. Answer: 47 Let \( \theta = \angle BAC \). Then \( 15 = 6 + |AC| + |BC| = 6 + 6 \cos(\theta) + 3(2\theta) \Rightarrow \theta = 46.514^\circ \).

6. Answer: 15 If only one color is used, there are 3 ways. If three faces are painted one color and one another color, there are 6 ways. If two faces are painted one color and the other two faces are painted a different color, there are 3 ways. If all three colors are used, there are 3 ways.

7. Answer: 99 \( x = |BC| \Rightarrow n = \sqrt{100^2 + x^2 - x} = 100^2/(\sqrt{100^2 + x^2} + x) \). This is a decreasing function of \( x \) with value 100 at \( x = 0 \). Thus there are 99 possible values for \( x \). Each such \( x \) is a positive rational.

8. Answer: 15 \( 1/a + 1/b + 1/c = (ab + ac + bc)/(abc) = (-120/5)/(-8/5) = 15 \).

9. Answer: 1673 \( 1 \leq a \leq 6 \Rightarrow a^6 \equiv 1 \pmod{7} \) and \( 1^n + 2^n + 3^n + 4^n + 5^n + 6^n \) is divisible by 7 for \( 1 \leq n \leq 5 \) but not \( n = 6 \). Thus, \( 1^n + 2^n + 3^n + 4^n + 5^n + 6^n \) is divisible by 7 if \( n \) is not a multiple of 6.

10. Answer: 2 \( 1/6 + 2/8 + 3/9 + 5/4 = 2 \), and \( \frac{a}{b} + \frac{c}{d} + \frac{e}{f} + \frac{g}{h} \geq \frac{a + c + e + g}{9} \geq \frac{10}{9} \).

**TIE BREAKER ANSWER:** \( \sum_{k=5}^{10} C(a,k)C(20,10-k)/C(20+a,10) > 0.95 \) for \( a > 41 \).