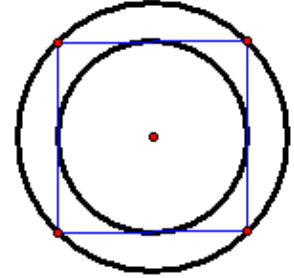


State Mathematics Contest Finals
GEOMETRY
April 30, 2009

1. A square is inscribed and circumscribed by two circles as shown. What is the ratio of the larger circle's circumference to the smaller circle's circumference?



- a) 2:1 b) $\sqrt{2} : 1$ c) $\sqrt{\pi} : 1$ d) 3:2
 e) Cannot be determined
2. Consider a right triangle with an area 120 cm^2 and a perimeter of 60 cm. What is the length of its hypotenuse?
- a) 20 cm b) 24 cm c) 26 cm d) $27\frac{1}{2}$ cm e) 30 cm
3. A room with dimensions $15 \text{ ft} \times 20 \text{ ft}$ and an 8 ft ceiling has a large ($84 \text{ in} \times 96 \text{ in}$) window unit and a door ($84 \text{ in} \times 36 \text{ in}$). Its walls need painting. Find the approximate (within nearest 100 ft^2) surface area of the walls?
- a) 500 ft^2 b) 600 ft^2 c) 400 ft^2 d) $2,400 \text{ ft}^2$ e) none of these

4. A rectangular field is partitioned into four rectangular parcels. The areas of three of these parcels are as shown. Determine the area of the field.

330 m^2	450 m^2
	600 m^2

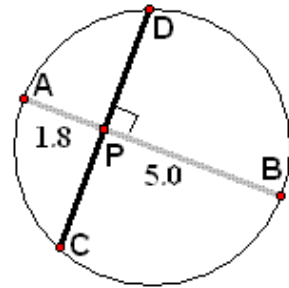
- a) 440 m^2 b) $1,800 \text{ m}^2$ c) $1,820 \text{ m}^2$
 d) $1,840 \text{ m}^2$ e) $8,190 \text{ m}^2$
5. Determine the next element in this sequence of ordered pairs:
 $\{(0, 1), (2, 1), (2, 3), (4, 5), (6, 9), (10, 15), \dots\}$.
- a) (16, 23) b) (16, 25) c) (14, 25) d) (14, 27) e) (16, 27)

6. Which of the following statements is NOT true about a right triangle?
- The smallest arc connecting the three vertices forms a semicircle.
 - The measure of the two smaller angles equals the larger angle.
 - All right triangles can be partitioned into two similar triangles.
 - Its area is twice the product of the lengths of the two shorter sides.
 - Its perimeter never exceeds $(1+\sqrt{2})$ times the length of its longest side.
7. You are asked to pull out (without replacement) three letters from a bowl containing the first ten letters of the alphabet, A – J. What is the chance that these letters rearrange to spell “BAD”?

- $\frac{3}{10}$
- $\frac{1}{1000}$
- $\frac{3}{500}$
- $\frac{1}{720}$
- $\frac{1}{120}$

8. Two perpendicular chords of a circle intersect at point P . If $AP = 1.8$ and $BP = 5.0$ determine CD .

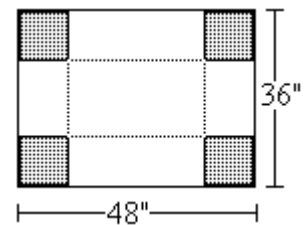
- 3
- 5.2
- 6.0
- 6.4
- None of these



9. Find the area of a right triangle whose two longest sides have lengths 24 and 25.

- 84
- 175
- 87.5
- 168
- 300

10. A rectangular 48” by 36” piece of cardboard is used to create an open box by cutting a square from each corner and folding up the sides. If one of the dimensions of the box is to be 20 inches what is its largest possible volume?



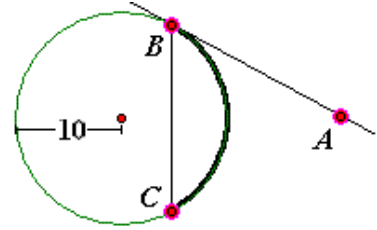
- $2,880 \text{ in}^3$
- $5,760 \text{ in}^3$
- $8,640 \text{ in}^3$
- $7,680 \text{ in}^3$
- $5,120 \text{ in}^3$

11. A 2 mm thick fabric is wrapped around a cardboard tube that has a 5 cm cross-sectional diameter. The outer diameter of the rolled up fabric is 25 cm. What is the approximate length of the rolled up piece of fabric?

- a) 5 m b) 30 m c) 47 m d) 24 m e) 16 m

12. A circle of radius 10 is cut by chord \overline{BC} . Line \overline{AB} is tangent to the circle at point B . If $m\angle ABC = 72^\circ$ what is the length of arc BC ?

- a) 0.4π b) 2π c) 4π d) 8π e) 12π



13. The surface area of a large spherical balloon is doubled. By what factor is the volume of the balloon increased?

- a) 8 b) 4 c) $2\sqrt{2}$ d) $\sqrt[3]{4}$ e) 2

14. What is the length of the hypotenuse of a right triangle that has one leg measuring 30 and an altitude (that is not a side) of measure $10\sqrt{5}$?

- a) $15\sqrt{5}$ b) 45 c) $20\sqrt{5}$ d) 50 e) none of these

15. A lighthouse is casting a 240 foot shadow. At the same time you notice the shadow of your 42 inch tall sister is 6 feet long. Assuming the ground is level and your sister is standing up straight, how tall is the light house?

- a) 140 ft b) 1,230 ft c) $411\frac{3}{7}$ ft d) 196 ft e) none of these

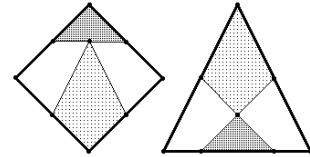
16. Find the equation of the line through point $(2, 1)$ and tangent to the circle, $(x-3)^2 + (y-4)^2 = 10$.

- a) $y = 3x - 5$ b) $y = -3x + 7$ c) $y = \frac{x+1}{3}$ d) $y = \frac{-x+5}{3}$ e) none of these

17. Consider a pentagon with two right angles and the remaining interior angles all equal. Determine the measure of the three non-right angles.

- a) 60° b) 72° c) 108° d) 144° e) none of these

18. A unit square is divided into 4 pieces using three cuts as shown. The four pieces can be rearranged to form an isosceles triangle. What is the length of the longest side of the triangle?

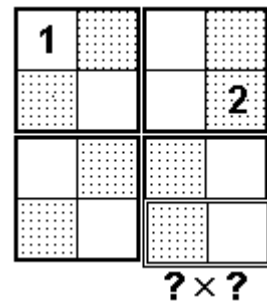


- a) $\frac{2}{\sqrt[4]{3}}$ b) $\sqrt{2}$ c) $\frac{\sqrt{5}}{2}$ d) $\frac{\sqrt{5}}{\sqrt{2}}$ e) $\sqrt{5}$

19. Find the area of a triangle that has points, (1, 7), (6, 2) and the origin as its vertices.

- a) 20 b) 21 c) 12.5 d) $5\sqrt{2}$ e) none of these

20. Consider this modified Sudoku puzzle. The numbers 1 through 4 appear in every column, every row, and every 2×2 exactly once. In addition a shaded square will contain an even number, while an un-shaded square will contain an odd one. Find the product of the values in the two right-most squares of the bottom row.



- a) 2 b) 4 c) 6 d) 8 e) 12

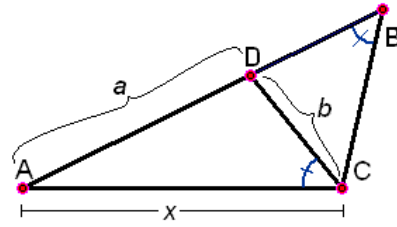
21. Six chords are used to partition a disk. What is the maximum number of partitions that can be obtained by this process?

- a) 16 b) 21 c) 22 d) 32 e) 64

22. A right circular cylindrical container has its height increased by 10% and its bases' circumference decreased by 10%. By what percentage did its volume change?

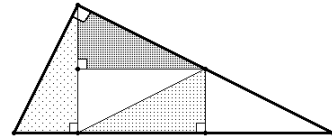
- a) 0% b) -10.9% c) -8.9% d) 2.8% e) $+8.9\%$

23. Consider the $\triangle ABC$ where \overline{CD} bisects $\angle ACB$.
 If $\angle ACD \cong \angle CBD$ and $AD = a$ and $CD = b$,
 determine AC .



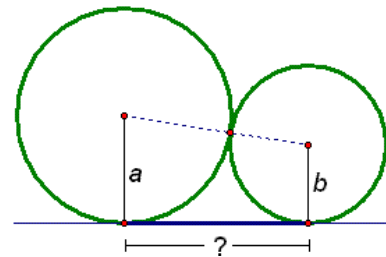
- a) $\sqrt{a^2 + ab}$ b) $\sqrt{a^2 + b^2}$ c) $\sqrt{a^2 + ab + b^2}$ d) $2\sqrt{ab}$ e) $\sqrt{(a-b)^2 + a^2}$

24. Consider a right triangle that can be partitioned into 5 congruent parts as shown. What is the ratio of the hypotenuse to its shortest leg?



- a) $\sqrt{2} : 1$ b) $\sqrt{3} : 1$ c) $2 : 1$ d) $\sqrt{5} : 1$ e) $(1 + \sqrt{5}) : 2$

25. Two mutually tangent circles, one with radius a the other with radius b , are both tangent to line. How far apart are the points of tangency?

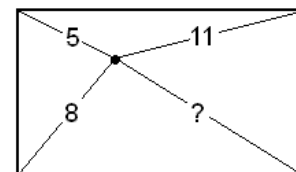


- a) $a + b$ b) $2\sqrt{ab}$ c) $2\sqrt{b^2 + a^2}$ d) $\frac{2ba}{a+b}$ e) $(\sqrt{a} + \sqrt{b})^2$

26. On an analog clock the two hands (hour and minute) form a 120° angle at 4 o'clock. How many minutes will pass when it is the next time that the two hands form a 120° angle?

- a) 60 min. b) 42 min. c) $65\frac{5}{11}$ min. d) $44\frac{1}{11}$ min. e) $43\frac{7}{11}$ min.

27. A rectangle has a point in its interior that is 5 units from one corner, 8 units from another and 11 units from the third. What is the distance of the point from the fourth corner?



- a) 14 b) 10 c) $4\sqrt{10}$ d) $\sqrt{82}$ e) none of these

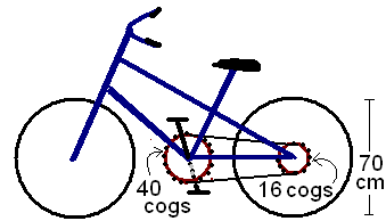
28. A piece of decorative ribbon is to be wrapped three times around a cylindrical container starting at the top and ending at bottom. If the container is 40 cm tall and has a diameter of 20 cm, what is the minimum length of the ribbon in centimeters?

- a) $40+60\pi$ b) $20\sqrt{13} \cdot \pi$ c) $20\sqrt{4+9\pi^2}$ d) 60π e) none of these

29. Harold forgot to write down a very important phone number. All he remembers is that it started with 713 and that the next set of 4 digits involved 1, 7 and 9 with one of these numbers appearing twice. If he guesses a phone number what is the chance that he gets the number correct?

- a) 1 in 36 b) 1 in 72 c) 1 in 24 d) 1 in 81 e) 1 in 27

30. Chris is riding her bicycle and wants to know how fast she is going. She is pedaling 80 turns in a minute. There are 40 cogs on the front sprocket and 16 cogs on the back wheel gear. If the wheel has a diameter of 70 cm and you use $22/7$ as an approximation of π , roughly how fast is she going?



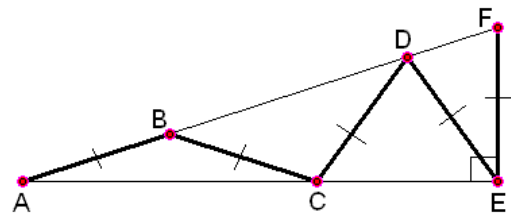
- a) 10.6 km/h b) 26.4 km/h c) 42.2 km/h d) 44.0 km/h e) none of these

31. A right triangle with area 330 cm^2 has as its shortest side of length 11 cm. What is the length of its perimeter?

- a) 132 cm b) 60 cm c) 121 cm d) 144 cm e) none of these

32. Consider the right triangle with points on its sides as shown. If $AB=BC=CD=DE=EF$ determine $m\angle FAE$.

- a) 10° b) 15° c) 18°
 d) 20° e) $22\frac{1}{2}^\circ$



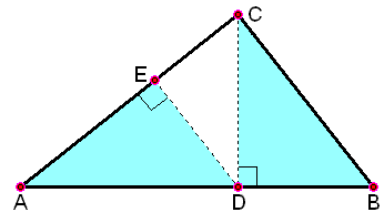
33. An aquarium measuring $50\text{cm} \times 30\text{cm} \times 40\text{cm}$ holds 132 lbs of water. If each dimension of the tank is increased by 10cm, what is the weight of the water if that tank is filled?

- a) 120 lbs b) 162 lbs c) 171.6 lbs d) 264 lbs e) none of these

34. The interior angles of a pentagon have the following measures: $x^\circ, x^\circ, 2x^\circ, 3x^\circ, 5x^\circ$. Determine x .

- a) 30 b) 45 c) 60 d) 72 e) none of these

35. The triangle $\triangle ABC$ with altitude \overline{CD} and point E on side \overline{AC} has $\triangle ADE \cong \triangle CBD$. By what factor is the area of $\triangle ABC$ larger than the area of $\triangle ADE$?

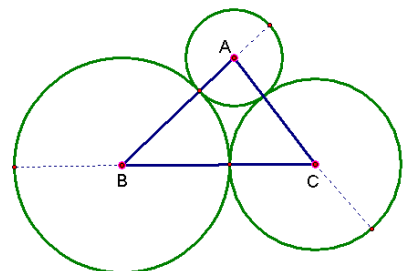


- a) 2.5 b) $\sqrt{5}$ c) $\frac{1+\sqrt{5}}{2}$
 d) $1+\frac{\sqrt{5}}{2}$ e) $\frac{3+\sqrt{5}}{2}$

36. George, lost in the dessert, starts at point A travels 4 km north, then turns and continues 3 km east, then turns southwest and continues $\sqrt{8}$ km and ends at point B. How far is point A from point B?

- a) 3 km b) $(5-\sqrt{8})\text{km}$ c) $\sqrt{13}$ km d) $\sqrt{17}$ km e) $\sqrt{5}$ km

37. $\triangle ABC$ has sides that measure 12, 9, 7. Three circles, each centered at one of the triangle's vertices are mutually tangent as shown. (Note that the image is not to scale.) Calculate the radius of the largest of the three circles.

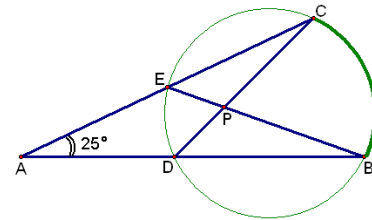


- a) 5 b) 6 c) $6\frac{1}{2}$ d) 7 e) $7\frac{1}{2}$

38. A triangle has two of its angles measuring 75° and 60° and its shortest side is 10 units. How many units does the longest side measure?

- a) $5 \cdot (1 + \sqrt{3})$ b) $5 \cdot (\sqrt{2} + 2)$ c) $5 \cdot (2 + \sqrt{3})$ d) $5 \cdot \sqrt{10}$ e) $5 \cdot (\sqrt{2} + \sqrt{6})$

39. Two rays emanating from point A intersect a circle twice as shown. If the arc BC covers 25% of the circle's circumference, and $m\angle BAC = 25^\circ$, calculate $m\angle DPB$.



- a) 105° b) 115° c) 145°
 d) 150° e) none of these

40. In the given alpha-numeric puzzle each letter has a one-to-one correspondence to a non-zero digit. The three words are numbers that form an addition problem. If $HINT = H18T$, find the value for "A".

$$\begin{array}{r}
 \mathbf{M A T H} \\
 \mathbf{I S N T} \\
 \hline
 \mathbf{H A R D}
 \end{array}$$

- a) 2 b) 3 c) 4 d) 6 e) 9