Algebra II
State Mathematics Contest Finals, April 28, 2011

1. Find the y value of the intersection of $y = 5x - 3$ and $7x - 2y = -3$.

   a) 1 b) 3 c) $\frac{6}{11}$ d) 7 e) 12

2. For what value of b will $|5x - 3| = 2x - b$ have a unique solution?

   a) 0.5 b) 0.6 c) 1 d) 1.2 e) 1.6

3. In a school where 60% of the students are boys, 30% of the students plan to go to college. If 45% of the girls want to go to college, what is the percentage of boys that want to go?

   a) 27% b) 20% c) 15% d) 12% e) none of these

4. Solve $25^x + 2 \cdot 5^x = 15$.

   a) 0 b) $\log_5 3$ c) $\log_3 5$ d) $\sqrt[5]{12}$ e) $\log_5 12$

5. What is the sum of all the positive integers between 1 and 2011 that are divisible by five?

   a) 81,003 b) 403,005 c) 404,215 d) 405,015 e) 810,030

6. A triangle is formed by the area bounded by the x- axis, $y = x$, and the line $y = b - 3x$. What is a value of b such that the area of the triangle is 10?

   a) $4\sqrt{15}$ b) $6\sqrt{10}$ c) $8\sqrt{15}$ d) 40 e) none of these

7. Which of the following expressions, if any, is different from the other 3? $\left(i = \sqrt{-1}\right)$

a) $i - \dfrac{4}{1+5i}$ b) $\dfrac{1+9i}{5-i}$ c) $\dfrac{23i-2}{13}$ d) $\dfrac{i-9}{1+5i}$ e) All are the same.

8. In a class with 14 women and 7 men, 2 students are chosen at random. If all have an equal chance of being picked, what is the chance of getting a pair (1 men and 1 women)?

a) $\dfrac{1}{2}$ b) $\dfrac{7}{30}$ c) $\dfrac{7}{15}$ d) $\dfrac{2}{9}$ e) $\dfrac{4}{9}$

9. If $A - B = 5$ and $A^2 + AB - 2B^2 = 10$, find the value of AB.

a) -4 b) -6 c) 6 d) -15 e) 45

10. Josh's average speed while driving from Charlotte to Asheville was 60 mph. The two cities are roughly 120 miles apart. If he drove 45 mph for 36 miles, how fast did he drive the remaining distance?

a) $66\frac{3}{7}$ mph b) 70 mph c) $72\frac{3}{7}$ mph d) 75 mph e) 82 mph

11. The function, $f(x)$, is defined as $f(x) + 2f(\dfrac{1}{x}) = x$ for all real numbers $x \neq 0$. Find $f(2)$.

a) -1 b) $-\dfrac{1}{3}$ c) 0 d) $\dfrac{1}{2}$ e) 1

12. On a circle O, we choose two points A and B. Find the probability that measure $\angle AOB$ is less than 60°.

a) $\dfrac{1}{12}$ b) $\dfrac{1}{6}$ c) $\dfrac{1}{3}$ d) $\dfrac{1}{2}$ e) 1

13. Let a, b, and c be integers satisfying $0 \le a < b < c \le 10$ and $c = a \times b$. How many different ordered triples (a, b, c) satisfy these conditions?

a) 2 b) 3 c) 4 d) 5 e) 6

14. Evaluate $\cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2 - \cdots}}}}}$

a) -1 b) $-\frac{1}{2}$ c) 0 d) $\frac{1}{2}$ e) 1

15. If $7^{-2x} = 3$, find the value of 7^{3x+1}.

a) 1 b) $\frac{7}{3}$ c) $\frac{7\sqrt{7}}{9}$ d) $\frac{7\sqrt{3}}{9}$ e) none of these

16. Let a, b, and c be distinct positive integers such that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$. Determine $a + b + c$.

a) 9 b) 10 c) 11 d) 12 e) 13

17. Let a and b be the solutions to the equation $x^2 + 3x + 1 = 0$. Find the value of $\frac{1}{a^2}+\frac{1}{b^2}$.

a) 1 b) 3 c) 5 d) 7 e) 8

18. If $f\left(\sqrt{\frac{8-x}{2+x}}\right) = x^2 + 3x$, find $f(2)$.

a) -2 b) -1 c) 0 d) 4 e) 10

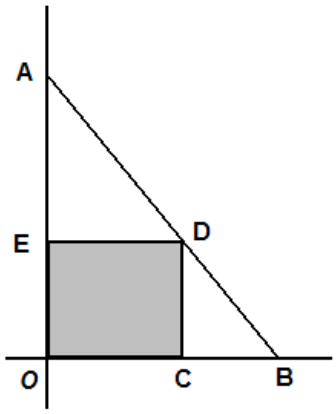
19. Sara and Kaleigh are painting a house together. If Sara works 8 days, Kaleigh will need 4 days to complete the job. If Sara works 4 days, Kaleigh will need 6 days to finish. How many days will Sara work if she paints the entire house by herself?

a) 8 days b) 10 days c) 12 days d) 14 days e) 16 days

20. Let $f(x) = \frac{1}{x(x+1)}$. Find $f(1) + f(2) + \cdots + f(2011)$.

a) $\frac{1005}{2011}$ b) $\frac{2010}{2011}$ c) $\frac{2011}{2012}$ d) $\frac{4047}{2012}$ e) $\frac{6066}{2011}$

21. Let point D be on the line segment connecting A and B as shown. If A is (0,8) and B is (4,0), what is the maximum area of the rectangle OCDE?



a) 2 b) 4 c) 6 d) 8 e) 10

22. For non-zero numbers x and y, let $f(xy) = f(x) + f(y)$ and $f(5) = 2$. Find $f(625)$.

a) 2 b) 4 c) 8 d) 12 e) 16

23. A positive two-digit integer is increased by 20% when its digits are reversed. Find the ones digit of the original integer.

a) 3 b) 5 c) 6 d) 7 e) 8

24. Let $(x+1)^5 = c_5x^5 + c_4x^4 + c_3x^3 + c_2x^2 + c_1x + c_0$. Find $c_2 + c_4$.

a) 0 b) 15 c) 16 d) 30 e) 32

25. A total of 6 people, consisting of 3 married couples, are randomly seated at a round table. What is the probability that all 3 couples are seated next to each other?

a) $\frac{2}{15}$ b) $\frac{1}{60}$ c) $\frac{1}{10}$ d) $\frac{2}{5}$ e) $\frac{3}{5}$

26. Find $x^2 - x$, where $x = \sqrt{5+\sqrt{5+\sqrt{5+\cdots}}}$.

a) 0 b) -5 c) 5 d) $-\sqrt{5}$ e) $\sqrt{5}$

27. Emma starts her trip 30 minutes before Paul. If she drives 60 mph and Paul drives 70 mph, how far will each have traveled when Paul catches up with Emma?

a) 3 miles b) 130 miles c) 175 miles d) 180 miles e) 210 miles

28. Find a rational solution to $2\cdot\log_x 3 = 2\cdot\log_3 x + 3$.

a) $\frac{1}{3}$ b) $\frac{1}{9}$ c) $\frac{1}{2}$ d) 9 e) no rational solution

29. Let $\frac{x}{y} = 0.6$ and $\frac{y}{z} = 0.75$. Find $\frac{x+z}{y}$.

a) $\frac{29}{15}$ b) $\frac{27}{20}$ c) $\frac{7}{8}$ d) $\frac{2}{3}$ e) none of these

30. If $3^x - 2\cdot(3)^{1-x} = 5$, find 3^{2x+1}.

a) 27 b) 36 c) 72 d) 81 e) 108

31. Three standard dice are rolled and the total of the top faces is 10. What is the probability that at least one die has the number "2" on its top face?

a) $\frac{4}{9}$ b) $\frac{5}{12}$ c) $\frac{12}{23}$ d) $\frac{91}{216}$ e) none of these

32. Let $g(x) = x^2 + x + 1$ and $f(x) = mx - 3$ where $m > 0$. Determine m such that $f(x) = g(x)$ has a unique solution.

a) $m = -3$ b) $m = 0$ c) $m = 3$ d) $m = 2.5$ e) none of these

33. Three relative primes when added together yield 42. The difference between the largest and smallest is 7. Determine the value of the middle number. (Relative primes are numbers with no common factors. In this case no two numbers of the three have a common prime factor.)

a) 11 b) 13 c) 15 d) 17 e) none of these

34. Find the radius of the circle centered at point (1, 1) that is tangent to the line $3x + 4y = 12$.

a) 1 b) 5 c) $\sqrt{13}$ d) $2\sqrt{1.7}$ e) none of these

35. The sides of a right triangle are c, c^2, and c^3, where $c > 1$. Calculate its area.

a) $\frac{1+\sqrt{5}}{2}$ b) $\frac{\sqrt{2}+\sqrt[4]{5}}{2}$ c) $\frac{\sqrt{2+\sqrt{5}}}{2}$ d) $\frac{\sqrt{2}+\sqrt{5}}{2}$ e) $\frac{5+\sqrt{2}}{2}$

36. Let $2^a + 2^b + 2^c + 2^d = 57$ where $a \neq b \neq c \neq d$ and $a, b, c, d \in \mathbf{Z}$. Determine $a + b + c + d$.

a) 9 b) 10 c) 11 d) 12 e) 13

37. Let $x - y = A$ and $x^2 - y^2 = B^2$. Assume $x \neq y$, and $x \neq -y$. Express x in terms of A and B.

a) $\frac{B^2 + A^2}{2}$ b) $\frac{\sqrt{B} + A}{2}$ c) $\frac{B^2 + A^2}{2A}$ d) $\frac{B^2 - A^2}{2A}$ e) $\frac{B^2 + A^2}{2AB}$

38. Solve $\log_x 12 = \log_x 75 - 2$ for x

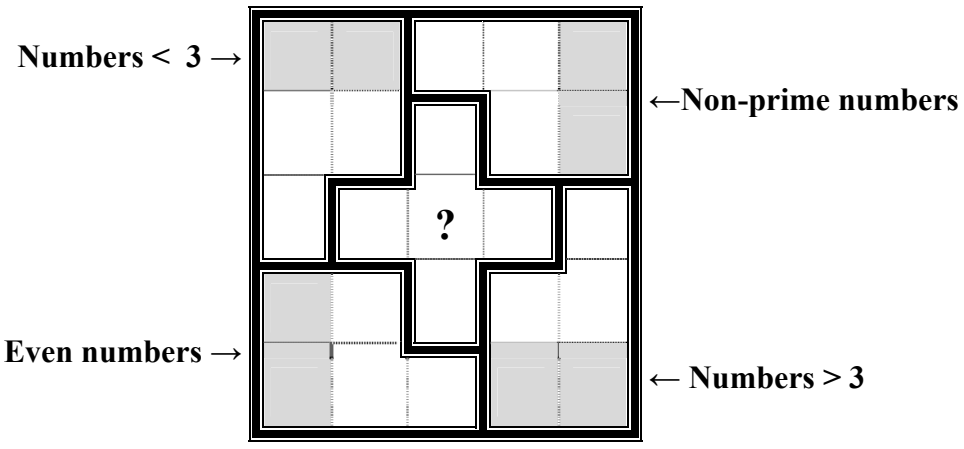
a) $\frac{5}{2}$ b) $\frac{4}{25}$ c) $\frac{2}{63}$ d) $\sqrt{32}$ e) $\sqrt[5]{2}$

39. Find all solutions to the equation, $\left| x^2 + x - 14 \right| = x + 11$, then calculate the sum of their absolute values.

a) 2 b) 14 c) 25 d) $10 + 2\sqrt{3}$ e) none of these.

40. The following is a 5×5 Sudoku puzzle. The numbers 1-5 appear exactly once in each row, each column, and each region. Along the outer cells that are shaded restrictions apply as indicated. Which of the numbers should be in the center cell? (Note, 1 is not a prime number.)

a) 1 b) 2 c) 3 d) 4 e) 5



Algebra II – Answer Key

April 28, 2011

1. E
2. D
3. B
4. B
5. D
6. A
7. E
8. C
9. A
10. B

11. B
12. C
13. B
14. E
15. D
16. C
17. D
18. A
19. E
20. C

21. D
22. C
23. B
24. B
25. A
26. C
27. E
28. B
29. A
30. E

31. A
32. E
33. B
34. A
35. C
36. D
37. C
38. A
39. B
40. D

Tie Breakers

Best of three: 15, 25, 35

Sudden Death: 40, 30, 20, 10, 33