

ALGEBRA I
2001 STATE MATH CONTEST FINALS

1. If $x^2 + y^2 = 25$ and $x^2 - y^2 = 7$ then $x^4 - y^4$ equals?
a) 7 b) 14 c) 49 d) 175 e) 337
2. If $f(x) = 3 - 2x$, calculate the value of $f^{-1}(5)$.
a) 3 b) -1 c) 2 d) 1 e) -2
3. Simplify $\frac{a^{-1}b^{-1}}{a^{-3} - b^{-3}}$.
a) $\frac{a^2b^2}{b^2 - a^2}$ b) $\frac{a^2b^2}{b^3 - a^3}$ c) $\frac{ab}{b^3 - a^3}$ d) $\frac{a^3 - b^3}{ab}$ e) $\frac{a^3 - b^3}{a - b}$
4. Which of the following functions is neither odd nor even?
a) $x^3 - 2x$
b) $x^3 - x|x|$
c) $x^5 - x^3 + 3x$
d) $x^4 - 2x^2 + |x + 1|$
e) $3x^2 - |x| + 6$
5. The coefficient of c^2d^2 in the expansion of $(\sqrt{c} + \sqrt{d})^8$ is:
a) 28 b) 56 c) 64 d) 70 e) 256
6. The capacity of a car's radiator is nine liters. The mixture of antifreeze and water is 30% antifreeze. The temperature is predicted to drop rapidly requiring the mixture to be 65% antifreeze. How much of the mixture in the radiator must be drawn off and replaced with pure antifreeze?
a) 3.15 liters b) 4.5 liters c) 5.0 liters d) 6.0 liters e) none of the these

7. If a , b , and c are numbers for which $\frac{x^2 + 10x - 36}{x(x-3)^2} = \frac{a}{x} + \frac{b}{x-3} + \frac{c}{(x-3)^2}$, then $a + b + c$ equals:
- a) 2 b) 3 c) 10 d) -36 e) 8
8. If $x + y = 11$ and $xy = 15$, find the value of $x^2 + y^2$.
- a) 91 b) 11 c) 136 d) 59 e) none of these
9. Which of the following sets does NOT represent a function?
- I $\{(x, y) \mid y = 2x + 1\}$
 II $\{(x, y) \mid x^2 + y^2 = 10, y \geq 0\}$
 III $\{(3,1), (4,1), (5,2), (6,2), (7,3)\}$
 IV $\{(x, y) \mid y = 2^x + 1\}$
 V $\{(x, y) \mid x^2 - y^2 = 2, x \geq 0\}$
- a) I b) II c) III d) IV e) V
10. If y varies jointly as x and the square of z and inversely as the cube of w , by what factor is y increased when z is doubled and w is divided by 3 if x remains the same?
- a) $\frac{4}{27}$ b) $\frac{27}{4}$ c) $\frac{1}{108}$ d) 108 e) 36
11. How much elevation does a car gain if it is going at 50 mph. for 15 minutes on a road that increases 6 feet in elevation for every 100 feet of driving?
- a) 2376 ft b) 4500 ft c) 3960 ft d) 5280 ft e) none of these
12. Find the sum of the solutions to the equation $x^2 - 3x = 10$.
- a) 3 b) 7 c) 10 d) 23 e) none of these
13. Find the solution to $|x| - 1 = 2x + 1$.
- a) -2 b) 2 c) -0.667823 d) $-\frac{2}{3}$ e) none of these

14. Mr. Gauss paid a total of \$623 (including tax) for two airline tickets, one for an adult and the other for a child. If the airline gives a 25% discount on the child's ticket and the airport charges a \$21 tax per ticket regardless of type, what is the total cost of the child's ticket (including the airport tax)?
- a) \$270.00 b) \$267.00 c) \$311.50 d) \$249.00 e) none of these
15. Given $(x - a)^2 = x^2 - a$, solve for x .
- a) $x = a$ b) $x = \sqrt{a}$ c) $x = \frac{a-1}{2}$ d) $x = \frac{\sqrt{a^2+1}}{2}$ e) $x = \frac{a+1}{2}$
16. Define \oplus as follows $a \oplus b = a^2 - ab + b^2$. Calculate $(2 \oplus 3) \oplus 4$.
- a) 11 b) 37 c) 147 d) 5 e) none of these
17. Define \oplus as before $a \oplus b = a^2 - ab + b^2$. Solve $7 \oplus x = 25 + 3x$.
- a) $x \in \{4, 6\}$ b) $x \in \{-9\}$ c) $x \in \{3, 7\}$ d) $x \in \left\{ \frac{7 \pm \sqrt{37}}{2} \right\}$ e) no solution
18. Find the x intercept of the line that passes through the points $(0, 7)$ and $(3, 5.5)$.
- a) 7 b) 19 c) 14 d) 12.5 e) none of these
19. Find the sum of all the solutions to the equation: $|3x - 10| = x + 6$.
- a) 1 b) 7 c) 8 d) 9 e) 10
20. Given $f(x) = \frac{2x+3}{x-1}$ and $g(x) = 2x-1$, find $f(g^{-1}(3))$.
- a) $\frac{9}{10}$ b) 2 c) $\frac{13}{4}$ d) $\frac{9}{2}$ e) 7

21. Solve the inequality $\frac{x^2 + x - 42}{x + 1} \geq 0$. Express your answer in interval notation.
- a) $[-7, -1) \cup [6, \infty)$
 b) $(-\infty, -7] \cup [6, \infty)$
 c) $[-\infty, -1) \cup [6, \infty)$
 d) $(-\infty, -7] \cup (-1, 6]$
 e) $(-\infty, -1) \cup (-1, \infty)$
22. If the average (arithmetic mean) of five consecutive integers is m , what is the product of the greatest and least of these five integers?
- a) 4 b) $(m+2)^2$ c) $2m$ d) $4m^2$ e) $m^2 - 4$
23. A ball is dropped from a height of 72 inches. The ball rebounds $\frac{1}{2}$ of the distance it falls on each bounce. How far has the ball traveled vertically when it ceases bouncing?
- a) 96 in. b) 144 in. c) 216 in. d) 288 in. e) more than 300 in.
24. Simplify $\frac{\frac{2}{x} + \frac{1}{2x}}{x + \frac{x}{2}}$.
- a) $\frac{5}{3x^2}$ b) $\frac{3}{x^2}$ c) $\frac{x}{4}$ d) $\frac{4x^2}{5}$ e) none of these
25. Simplify $\left(9^{3/2} - 8^{1/3}\right)^{3/4}$.
- a) 1 b) $9\sqrt[4]{3} + \sqrt[4]{8}$ c) $\sqrt{125}$ d) $11\frac{9}{50}$ e) none of these
26. The equation $\frac{x-3}{2} = \frac{1}{x-4}$ has two solutions. Their product is:
- a) 6 b) 7 c) 10 d) 12 e) 14

27. When the polynomial $x^4 - 3x^3 - 7x^2 + 7x + 2$ is divided by $x + 2$, the quotient is $x^3 + Bx^2 + Cx + 1$. The value of $B+C$ is:
- a) -8 b) -2 c) 0 d) 2 e) 10
28. Consider the following quadratic equations:
- I) $x^2 + 1 = 0$
 II) $x^2 - 2 = 0$
 III) $x^2 + x - 1 = 0$
 IV) $x^2 - x - 2 = 0$
- Which of these equations, if any, has two different real number solutions?
- a) II, III, and IV only
 b) II, and IV only
 c) I, II and III only
 d) I, II, III, IV
 e) none of these
29. A cottage at the beginning of the first year was worth \$25,000, but each year it lost ten percent of the value. Approximately how many years will pass before its value is reduced to \$2,500?
- a) 6.5 years b) 9 years c) 11 years d) 22 years e) 100 years
30. Baskins-Robbins offers 31 different flavors of ice cream. Suppose you wish to order a triple scoop, with each scoop a different flavor. How many different combinations of three flavors are possible?
- a) 4,495 b) 8,990 c) 13,485 d) 26,970 e) none of these
31. The sum of the roots of $x^4 - x^3 + 5x^2 + 4 = 0$ is:
- a) -1 b) 0 c) 1 d) 4 e) 5
32. If $(x + 3)$ is a factor of $x^3 - 2kx + k^2$, then k is:
- a) 3 b) -3 or 9 c) 3 or -9 d) -3 e) -9

33. Find the fourth term in an arithmetic sequence in which the second term is -4 and the eighth term is 32 .
- a) 4 b) 8 c) 10 d) 14 e) none of these
34. Let $x > 0$, simplify $\frac{\sqrt{x} + \sqrt{x+1}}{\sqrt{x} - \sqrt{x+1}}$.
- a) $-2x - 2\sqrt{x(x+1)} - 1$ b) $2x - 2\sqrt{x(x+1)} - 1$ c) $-2x + 2\sqrt{x(x+1)} - 1$
d) $2x + 2\sqrt{x(x+1)} + 1$ e) $-2\sqrt{x} - 1$
35. A triangle's sides have lengths $\frac{1}{x+2}$, $\frac{1}{x+1}$, and $\frac{1}{x}$, what are the possible values of x ?
- a) $0 < x < \sqrt{2}$ b) $x > 0$ c) $x > 1$ d) $x < 1$ e) $x > \sqrt{2}$
36. A wire x inches long is bent into the shape of a circle. Express the area of the circle (in square inches) in terms of x .
- a) $3.5^2 p$ b) $\frac{x^2}{2p}$ c) $\frac{x^2}{4}$ d) $\frac{x^2}{4p}$ e) none of these
37. A palindromic number is a number that does not change if its digits are reversed. How many different 3-digit palindromic numbers exist if zero is not used?
- a) 18 b) 72 c) 81 d) 144 e) 729
38. If the three points $(-2, -1)$, $(x, 2)$, and $(8, 14)$ are collinear, then the value of x is:
- a) 1 b) 1.5 c) 2 d) 3 e) none of these
39. The distance from the point $(3, 10)$ to the line $y = 2x + 3$ is:
- a) $\frac{\sqrt{5}}{5}$ b) $\frac{2\sqrt{5}}{5}$ c) 1 d) $\sqrt{5}$ e) none of these
40. The solution of the equation $2^{x+3} = 4^{2x+1}$ is:
- a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{2}{3}$ d) 1 e) 2