

State Mathematics Algebra II Contest

May 3, 2001

1. If the length and width of a rectangle were increased by 1, the area would be 84. The area would be 48 if the length and width were diminished by 1. Find the perimeter P of the original rectangle.
- a. $10 < P < 20$ b. $20 < P < 30$ c. $30 < P < 40$
d. $40 < P < 50$ e. none of the above
2. In the sequence $\dots, w, x, y, z, 0, 1, 1, 2, 3, 5, 8, \dots$, each term is the sum of the two terms to its left. Find the value of $2 \cdot (w + x + y + z)$.
- a. 2 b. 1 c. 0
d. -1 e. -2
3. Simplify: $\sqrt{\frac{2^{x+4} - 2(2^{x+1})}{2(2^{x+3})}}$
- a. $\frac{3}{8}$ b. $\frac{\sqrt{3}}{4}$ c. 2^x
d. $\frac{x\sqrt{3}}{4}$ e. $\frac{\sqrt{3}}{2}$
4. A coin is biased so that the probability of obtaining a head is 0.25. Another coin is biased where the probability of obtaining a head is 0.6. If both coins are tossed, find the probability of obtaining at least one head.
- a. $\frac{11}{20}$ b. $\frac{7}{10}$ c. $\frac{3}{10}$
d. $\frac{9}{10}$ e. none of the above
5. If a polynomial, $F(x)$, has real coefficients with zeros at $2, 1 + i, 3 - i$, then this polynomial must have a degree of :
- a. at least 5 b. exactly 6 c. exactly 3
d. at least 6 e. none of the above

29. Solve $\frac{e^x - e^{-x}}{2} = 2$ for x , where $x > 0$.
- a. $\ln(2 + \sqrt{5})$ b. $\ln(2 \pm \sqrt{5})$ c. $\ln 2 + \ln \sqrt{5}$
d. 4 e. none of the above
30. If a , b , and c are integers such that $(\sqrt[3]{4} + \sqrt[3]{2} - 2)(a\sqrt[3]{4} + b\sqrt[3]{2} + c) = 20$, find the value of $a + b - c$.
- a. 10 b. 18 c. 6
d. $2\sqrt[3]{4}$ e. $\sqrt[3]{4}$
31. Find the sum of all proper fractions whose denominators are less than or equal to 100. (Include unreduced fractions in the sum.)
- a. 2075 b. 1050 c. 1175
d. 1275 e. 2475
32. Given a right triangle with sides of length a , b , and c and area, $a^2 + b^2 - c^2$. Find $\frac{c}{b}$, the ratio of the legs of the right triangle.
- a. 1 b. $\frac{\sqrt{3}}{2}$ c. 4
d. $\frac{1}{4}$ e. none of the above
33. Lynn purchased two candles of equal length. One of the candles will burn up completely in 5 hours, while the other candle requires 7 hours to burn up completely. If the candles are lit at the same time, approximately how long will they burn before one of the candles is twice the length of the other?
- a. 3.9 hr b. 3.2 hr c. 2.8 hr
d. 3 hr e. none of the above
34. A polynomial $P(x)$ has remainder -5 when divided by $x + 1$ and remainder 7 when divided by $x - 5$. What is the remainder when $P(x)$ is divided by the product of $x + 1$ and $x - 5$?
- a. $2x + 3$ b. $2x - 3$ c. $3x - 2$
d. 35 e. -35

