

**State Mathematics Contest: Geometry**  
May 3, 2001

1. Find the area of an equilateral triangle whose vertices lie on a circle with radius 2 cm.
- a.  $3p$  cm<sup>2</sup>      b.  $3\sqrt{3}$  cm<sup>2</sup>      c.  $\sqrt{3}p$  cm<sup>2</sup>      d. 6 cm<sup>2</sup>      e. none of these

2. The following statements describe a race between: Pat, Chris, Jo, and Sam.
- i. Pat is 12 seconds behind the next runner.
  - ii. The leader is 20 seconds ahead of the last person.
  - iii. Chris is 1 second ahead of somebody.
  - iv. Sam is 19 seconds ahead of Jo.

Which of the following gives the order of the runners. (Runners listed from first to last.)

- a. Sam, Chris, Pat, and Jo
  - b. Chris, Sam, Pat, and Jo
  - c. Sam, Pat, Chris, and Jo
  - d. Chris, Sam, Jo, and Pat
  - e. Sam, Chris, Jo, and Pat
3. If a boat travels North for 5 miles then East for 12, then Southeast for 6, approximately how far is it from its starting point?
- a. 13 miles      b. 19.01 miles      c. 23.21 miles      d. 16.26 miles      e. none of these

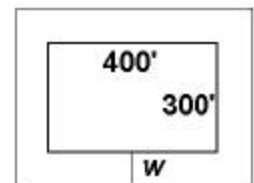
4. Given the following two circles, find the algebraic equation of the chord they share in common.

Circle I:  $(x-1)^2 + (y+2)^2 = 9$

Circle II:  $(x+3)^2 + (y-1)^2 = 16$

- a.  $4x - 3y = 1$       b.  $3x + 4y = 1$       c.  $2x - y = 10$       d.  $x - 3y = 5$       e. none of these

5. The area of a 300' by 400' rectangle is doubled by adding a strip of width "w" around the perimeter. Approximately how wide is that strip?



- a. 87.50'      b. 83.41'      c. 72.84'      d. 71.22'      e. none of these

6. Find the length of the perimeter of a right triangle whose area is  $30 \text{ cm}^2$  and whose hypotenuse is 13 cm.

- a.  $23 + \sqrt{69}$  cm    b.  $16 + 2\sqrt{34}$  cm    c. 30 cm    d.  $7 + 3\sqrt{13}$  cm    e. none of these

7. Find the area of a triangle whose vertices are (0,0), (12, 9) and (14, 6).

- a. 27 sq units    b. 54 sq units    c. 84 sq units    d. 168 sq units    e. none of these

8. Given that it is 3 o'clock, exactly how long will it take for the minute hand to catch up with the hour hand?

- a. 15 min    b. 16 min    c.  $16\frac{1}{3}$  min    d.  $16\frac{1}{2}$  min    e. none of these

9. If the size of a rectangle's area is twice as large as the size of its perimeter, and the length of one of its sides is 4.5, what is the length of the other side?

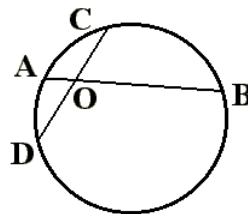
- a. 36    b. 3.6    c. 18    d. 16    e. none of these

10. A given circular cylindrical can is made up of a square piece of metal and two circular disks each with diameter  $c$  inches. What is the volume of this cylindrical can?

- a.  $2p \cdot c^3 \text{ in}^3$     b.  $\frac{p^2 c^3}{4} \text{ in}^3$     c.  $\frac{p^3 c^3}{6} \text{ in}^3$     d.  $2.467 c^3 \text{ in}^3$     e. none of these

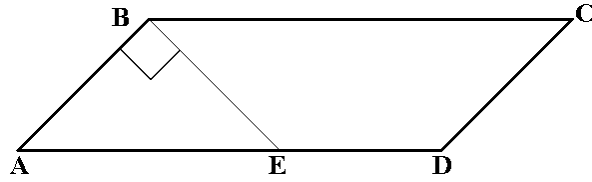
11. Two chords  $\overline{AB}$  and  $\overline{DC}$  intersect each other so that  $AO = 1.2$ ,  $OB = 7.5$ , and  $OC = DO$ . How long is  $\overline{DC}$  ?

- a. 6 units  
 b.  $\sqrt{8.7}$  units  
 c.  $\sqrt{35}$  units  
 d. 8.7 units  
 e. none of these



12. Find the area of parallelogram ABCD, given that  $AB = BE = ED = 1$ , and  $\angle ABE = 90^\circ$ .

- a.  $\sqrt{2}$
- b. 1.707
- c.  $\frac{2 - \sqrt{2}}{2}$
- d.  $\frac{2 + \sqrt{2}}{2}$
- e. none of these

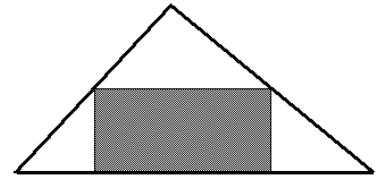


13. Find the solution set for  $y$  given that  $x, y, z$  are natural numbers and the following are true:

- i. If  $y < 3$  then  $x < 3$
- ii.  $z + y < 7$
- iii.  $x + z > 10$ .

- a.  $y \in \{1,2\}$
- b.  $y \in \{1,2,\dots,5\}$
- c.  $y \in \{3,4,5,6\}$
- d.  $y \in \{3,4,5\}$
- e.  $y \in \{1,2,\dots,6\}$

14. A rectangle is inscribed in a triangle such that its upper right vertex bisects the triangle's side. What is the ratio of the area of the shaded region to the area of the unshaded region?



- a. 1:2
- b. 4:3
- c. 5:4
- d. 2:3
- e. none of these

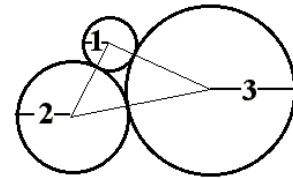
15. Two 5-12-13 triangles are combined to form a parallelogram. Which of the following statements must be true?

- a. The parallelogram is a rectangle.
- b. The perimeter of the parallelogram is 34 units long.
- c. The area of the parallelogram is 60 square units.
- d. One of the sides of the parallelogram is 12 units long.
- e. All of these statements must be true.

16. To construct a circle that circumscribes a triangle one finds its center by locating the intersection of which two lines.

- a. The perpendicular bisectors of two sides.
- b. The bisectors of two of the angles.
- c. Two medians.
- d. Two altitudes.
- e. It is not possible to construct a circle that circumscribes a triangle.

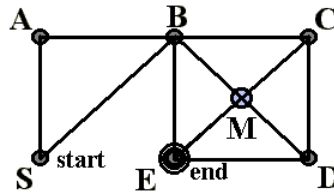
17. Form a triangle by connecting the centers of three circles. Each of these circles is tangent to the other two and their radii are 1, 2, and 3 units long. How large is area of the triangle?



- a. 12                      b.  $\sqrt{5} \cdot p$                       c. 24                      d.  $\sqrt{17}$                       e. 6

18. Given the graph below, how many different paths are there from "S" to "E" if one never visits the same point twice?

- a. less than 11  
b. 11  
c. 12  
d. 13  
e. more than 13



19. A pentagon is inscribed in a circle of radius 7. How long is the circular arc that connects two neighboring vertices?

- a.  $1.4p$                       b.  $2.1p$                       c.  $2.8p$                       d.  $7p$                       e. none of these

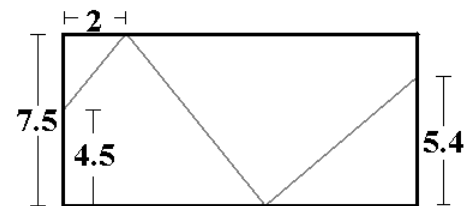
20. If a rectangle has a diagonal of length  $c$  and a perimeter of length  $p$  then the expression for its area is?

- a.  $pc$                       b.  $\frac{p^2 - 4c^2}{8}$                       c.  $\frac{\sqrt{p^2 + 2c^2}}{2}$                       d.  $\frac{p^2 + c^2}{2}$                       e. none of these

21. A rectangular solid with a square base has dimensions  $5 \times 5 \times 8$ . If its volume is quadrupled by doubling the lengths of sides of the base, by what factor is its surface area increased?

- a. 4                      b. 2                      c.  $\sqrt[3]{4}$                       d.  $2\frac{10}{21}$                       e. none of these

22. How long is the room shown in the figure on the right, given that it has a 7.5 foot ceiling, and an infra red beam that starts at one end of the room is bounced off the ceiling then the floor and finally hits the opposite side of the room 5.4 feet above the floor? The point at which it bounces off the ceiling is two feet from the wall.



- a. 8.6 ft                      b. 10.6 ft                      c.  $(2 + \sqrt{72})$  ft                      d. 12 ft                      e. none of these

23. Find the height of a square pyramid formed by four equilateral triangles whose sides all have length 2.
- a. 1                      b.  $\frac{\sqrt{6}}{2}$                       c.  $\sqrt{2}$                       d.  $\sqrt{3}$                       e. none of these
24. If a rectangle whose length is 9 times its width is modified so that its area is doubled but its perimeter is kept constant, what is the ratio of length to width for the new rectangle?
- a.  $5 + \sqrt{7} : 5 - \sqrt{7}$       b. 2:1                      c.  $7 + \sqrt{5} : 7 - \sqrt{5}$       d. 1:1                      e.  $\sqrt{6} + 1 : \sqrt{6} - 1$
25. One of the five statements below is false and the other four are true. Who told the falsehood?  
 Ann said, "If the car was not locked then the wallet was stolen."  
 Bo said, "If the tickets to the game are lost then Ann's statement is false."  
 Chris said, "Ann's statement is true"  
 Dan said, "The tickets to the game are not lost."  
 Ed said. "The wallet was stolen but not the tickets to the game."
- a. Ann                      b. Bo                      c. Chris                      d. Dan                      e. Ed
26. A Quadrilateral  $ABCD$  is inscribed in a circle. If the size of the angle at vertex  $A$  is  $36^\circ$  then the angle at vertex  $C$  is:
- a.  $72^\circ$                       b.  $54^\circ$                       c.  $126^\circ$                       d.  $144^\circ$                       e. none of these
27. The following facts are given: a gallon of paint covers 400 square feet of wall space, the room to be painted has an 8 foot ceiling, its dimensions are 20 by 14 feet, it has two doors (36 by 84 inches) and four large windows (72 by 60 inches), and you need two coats of paint. How much paint do you need to paint the walls?
- a. 1.47 gal.                      b. 2.06 gal.                      c. 2.72 gal                      d. 1.03 gal.                      e. none of these
28. For the points (0, 2), (6, 6) and (10, 0) which of the following statements are true?
- I. The points form the vertices of a right triangle.  
 II. The points form the vertices of an isosceles triangle.  
 III. The largest angle is at the vertex located on (6,6).
- a. only III                      b. II and III                      c. I and III                      d. all are true                      e. none are true

29. Given you have 4 sticks, two of length 5 and two of length 8, with which you are to form a quadrilateral. If at least one of the angles is a right angle, how many different noncongruent quadrilaterals could you form.

- a. 1                      b. 2                      c. 3                      d. 4                      e. more than 4

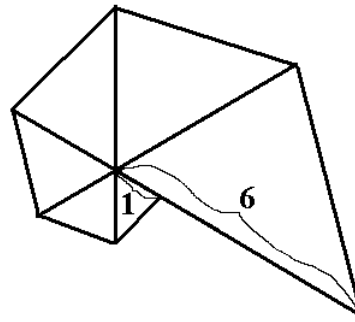
30. A triangle with sides 6, 8, and 10 has its shortest side doubled in length while the other two sides remain the same. What is the area of the new triangle?

- a. 30                      b. 40                      c.  $15\sqrt{7}$                       d.  $8\sqrt{30}$                       e. none of these

31. What is the size of an angle between two adjacent sides of a regular 12 sided polygon?

- a.  $144^\circ$                       b.  $120^\circ$                       c.  $\frac{1080^\circ}{7}$                       d.  $30^\circ$                       e. none of these

32. As shown in the figure on the right six similar triangles are each sharing one side with the next triangle and all are sharing one vertex. All angles at that vertex measure  $60^\circ$ . If the side of the last (smallest) triangle that is adjoining the first triangle is  $\frac{1}{6}$  as large as the longest side of that first triangle, how many times larger is the area of the largest triangle as compared to the smallest?



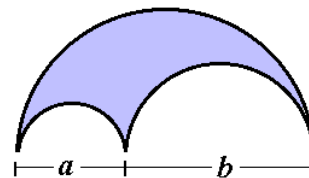
- a. 6                      b.  $\sqrt[6]{6^5}$                       c.  $\sqrt[3]{6^5}$                       d. 36                      e. none of these

33. Three views of the same block are shown on the right. What letter is on the side parallel to the side with the letter A?



- a. E                      b. O                      c. S                      d. H                      e. G

34. An *arbelos* is the region formed by three mutually tangent circles whose centers are colinear, as noted in the image. If the diameters of the two smaller circles are  $a$  and  $b$ , what is the area of the *arbelos*.



- a.  $abp$                       b.  $\frac{\sqrt{abp}}{4}$                       c.  $\frac{abp}{4}$                       d.  $\frac{\sqrt{a^2 + b^2}p}{4}$                       e. none of these

35. A bicycle has a 70 cm diameter wheel. If you ride in a 120 km race, approximately how many revolutions does the wheel have to make to complete the race?

- a. 54,600      b. 171,400      c. 538,600      d. 732,100      e. none of these

36. If the volume of a tetrahedron is doubled without changing its shape, by what factor is the surface area increased?

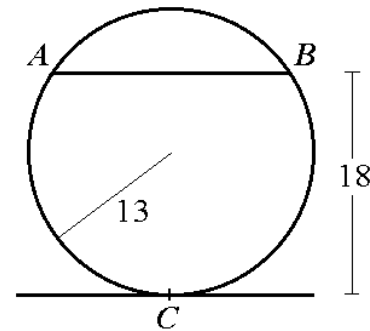
- a.  $\sqrt[3]{2}$       b.  $\sqrt[3]{4}$       c. 2      d.  $\sqrt{8}$       e. 4

37. A semicircle with diameter 12 inches is used to form a conical cup by bending the semicircle so that its two corners are connected and the circle's center forms the point of the cone. How large is the volume?

- a.  $9p\sqrt{3}$       b.  $27p\sqrt{3}$       c.  $18p\sqrt{6}$       d.  $2p\sqrt{15}$       e. none of these

38. Chord  $\overline{AB}$  is parallel to the line tangent to the circle at point  $C$ . If the distance between the chord and the tangent line is 18, and the radius of the circle is 13, how long is the chord  $\overline{AB}$ ?

- a. 12  
 b.  $2\sqrt{105}$   
 c.  $2\sqrt{155}$   
 d. 24  
 e. none of these



39. A rectangle with dimensions 11 by 13 had its diagonal increased by 50% without lengthening the shorter side. Approximately how big is the area of the new rectangle?

- a. 214.5      b. 321.75      c. 285.852      d. 175.139      e. 253.597

40. Nikki sees that the top of a 15 foot lamp, which is 250 feet away, lines up perfectly with the peak of a distant mountain. Nikki knows that the mountain is 15 miles away so she uses the lamp to determine the mountain's height. If Nikki's eyes are 5 feet above the ground, what is the best estimate of the mountain's height relative to Nikki?

- a. 900 feet      b. 3,178 feet      c. 4,752 feet      d. 5,625 feet      e. 6,648 feet