1999 Algebra I State Finals

1. If $\frac{x-y}{x+y}=\frac{5}{2}$, then $\frac{x}{y}$ is equal to

   a. $\frac{-7}{3}$ b. $\frac{7}{3}$ c. 3 d. $-\frac{7}{5}$ e. $-\frac{5}{7}$

2. A line contains (2, –7) and (–4, 2). Which statement is not true?

   a. The line has y-intercept –4
   b. The line contains $\left(1, \frac{-11}{2}\right)$
   c. The line is parallel to 3x– 2y = 6
   d. The line is perpendicular to $y=\frac{2}{3}x+4$
   e. No portion of the graph of the line lies in the first quadrant.

3. In your search for a summer job, you are given the following offers.

   Offer 1: At Timmy's Tacos you will earn $4.50 an hour. However, you will be required to purchase a uniform for $45.00. You will be expected to work 20 hours each week.
   Offer 2: At Kelly's Car Wash you will earn $3.50 an hour. No special attire is required. You must agree to work 20 hours each week.

   Before deciding which job offer you wish to take, you consider many factors. Which conclusion below is not true?

   a. If I work 8 weeks at Kelly's Car Wash and save all my earnings, I'll be able to save $560.
   b. If I take the job at Timmy's Tacos, I'll have to work 10 hours just to pay for purchasing my uniform.
   c. If I only plan to work for two weeks, I should choose the job at Kelly's Car Wash.
   d. If I plan to work for at least 3 weeks, I should choose the job at Timmy's Tacos.
   e. The job at Timmy's Tacos pays more if I work more than forty hours.

4. The operation # is defined as: $x \# y=\frac{x^2-y^2}{x+y}$

   If $x \# 7=3$, then which of the following is a possible value of x?

   a. –10 b. –7 c. –4 d. 3 e. none of these

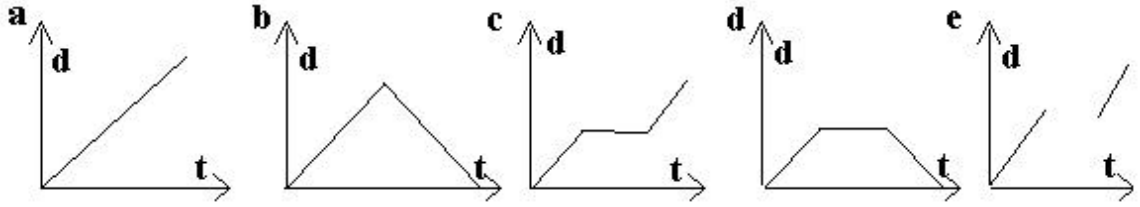
5. The number $2^n + 2^n + 2^n + 2^n$ can be written as

   a. 2^{4n} b. 16^n c. $24(4n^2-2)$ d. 2^{2n+2} e. $16^{\frac{n+2}{4}}$

6. If a is any real number, for what real value(s) of b does the equation $|x + a| = b$ have NO solutions for x?

a. All $b < 0$ b. Only $b = -1$ c. Only $b = 0$ d. All $b \neq 0$ e. All $b > 0$

7. Starting at her doorstep. Ramona walked down the sidewalk at 1.5 feet per second for 4 seconds. Then she stopped for 4 seconds, realizing that she had forgotten something. Next she returned to her doorstep along the same route at 1.5 feet per second. The graph of Ramona's distance (d) from her doorstep as a function of time (t) would most resemble which of the following?



Mientka Publishing Company prices its best seller *Where's Walter?* as follows:

$$C(n)=\begin{cases}12n, & \text{if } 1\le n\le 24,\\ 11n, & \text{if } 25\le n\le 48,\\ 10n, & \text{if } 49\le n,\end{cases}$$

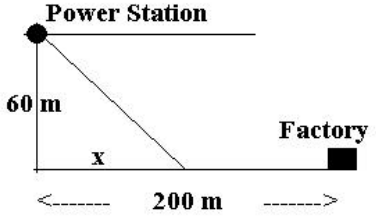
where n is the number of books ordered, and $C(n)$ is the cost in dollars of n books. Notice that 25 books cost less than 24 books. For how many values of n is it cheaper to buy more than n books than to buy exactly n books?

a. 3 b. 4 c. 5 d. 6 e. 8

8. For the linear function $f(x) = mx + k$, $f(1) = -2$ and $f(-2) = 11$. The change in the value of $f(x)$ when x increases by 2 is

a. $\frac{-13}{3}$ b. $\frac{-26}{3}$ c. –12 d. –13 e. $\frac{-39}{2}$

10. A power station and a factory are on opposite sides of a river 60 m wide, as shown. A power line must be run from the station to the factory. It costs \$25 per meter to run the cable in the river and \$20 per meter on land. Let x represent the distance downstream from the power station to the point where the cable touches land, as shown. The total cost of running the power line is the least for which of the following values of x?



a. 65 b. 70 c. 75
d. 80 e. 85

11. How many integers are there such that $7x + 2 \leq 23$ and $3x - 5 \geq 1$?

a. 0 b. 1 c. 2 d. 3 e. more than 3

12. If $x^3 - 3x + 7$ is divided by $x^2 - x + 1$, the remainder is

a. $x + 1$ b. $-5x + 8$ c. $-3x + 6$ d. $-2x + 6$ e. $-x + 8$

13. An operation $*$ is defined so that $1*3 = 5$, $6*9 = 21$, and $8*2 = 18$. The value of $11*20$ is

a. 39 b. 42 c. 48 d. 51 e. 59

14. The following instructions to a computer are carried out in the order specified.

1. LET S = 0
2. LET X = 5
3. LET THE NEW VALUE OF S EQUAL THE OLD VALUE OF S PLUS THE VALUE OF X
4. INCREASE THE VALUE OF X BY 2
5. IF X < 8, GO BACK TO INSTRUCTION 3. OTHERWISE GO ON TO INSTRUCTION 6
6. WRITE THE FINAL VALUE OF S

What value of S should be written in instruction 6?

a. 0 b. 5 c. 7 d. 9 e. 12

15. When a polynomial $P(x)$ is divided by $x-2$, the quotient is $3x^2 - x - 5$ and the remainder is -1. Then $P(-1)$ equals

a. 2 b. 3 c. 4 d. 7 e. 9

16. If *p* and *r* are different prime numbers greater than 9, which of the following must be true?

I. $\frac{p}{r}$ is less than 1.
II. $p + r$ is not prime.
III. *p* x *r* has three different positive integer factors greater than 1.

a. none b. III only c. I and II d. I and III e. II and III

17. A certain 90-mile trip took 2 hours. Exactly $\frac{1}{3}$ of the distance traveled was by rail, and this part of the trip took $\frac{1}{5}$ of the travel time. What was the average rate, in miles per hour of the rail portion of the trip?

a. 12 mph b. 30 mph c. 45 mph d. 60 mph e. 75 mph

18. For all positive integers x, let $\boxed{x}$ be defined as the sum of the integers from 1 to x, inclusive. Which of the following equals $\boxed{6} - \boxed{5}$?

a. $\boxed{1}$ b. $\boxed{2}$ c. $\boxed{3}$ d. $\boxed{5}$ e. $\boxed{6}$

19. Of the following statements, the one that is false is:

a. The set of integers is closed under multiplication.
b. The set of natural numbers is closed under addition.
c. The set of irrational numbers is closed under multiplication.
d. The set of rational numbers is closed under subtraction.
e. None of these.

20. If $r \in \{3, 6\}$ and $s \in \{-1, 2\}$, the largest value of $3r - s$ is:

a. 16 b. 19 c. 7 d. 10 e. None of these

21. A square has an area of R^2. An equilateral triangle has a perimeter of E. If r is the perimeter of the square and e is a side of the equilateral triangle, then, in terms of R and E, $e + r =$

a. $\frac{E+R}{7}$ b. $\frac{4R+3E}{3}$ c. $\frac{3E+4R}{12}$ d. $\frac{12E+R}{3}$ e. $\frac{E+12R}{3}$

22. Given that $r \neq 0$ and $r = 5w = 7a$, find the value of $r - w$ in terms of a.

a. $\frac{1a}{7}$ b. $\frac{7a}{5}$ c. $3a$ d. $\frac{28a}{5}$ e. $28a$

23. If p and q are positive integers, x and y are negative integers, and if $p > q$ and $x > y$, which of the following must be less than zero?

I. $q - p$
II. qy
III. $p + x$

a. I only b. III only c. I and II only d. II and III only e. I, II, and III

24. If the sum of 5 consecutive positive integers is w, in terms of w, which of the following represents the sum of the next 5 consecutive positive integers?

a. $w + 5$ b. $5w + 5$ c. $5w + 25$ d. $w + 25$ e. $w^2 + 25$

25. For distinct real numbers x and y, let $M(x, y)$ be the larger of x and y and let $m(x, y)$ be the smaller of x and y. If $a < b < c < d < e$, then $M(M(a, m(b, c)), m(d, m(a, e)))$ equals

a. a b. b c. c d. d e. e

26. Let p be an odd whole number and let n be any whole number. Which of the following statements about the whole number ($p^2 + n \cdot p$) is always true?

    a. It is always odd.
    b. It is always even.
    c. It is even only if n is even.
    d. It is odd only if n is odd.
    e. It is odd only if n is even.

27. Find the product of all real solutions of $16^{x^2+x+4} = 32^{x^2+2x}$.

    a. –8 b. 4 c. –16 d. –4 e. none of these

28. Let c be a constant. The simultaneous equations $x - y = 2$
    $cx + y = 3$

    have a solution (x, y) inside quadrant I if and only if

    a. $c = -1$ b. $c > -1$ c. $c < 3/2$ d. $0 < c < 3/2$ e. $-1 < c < 3/2$

29. An experiment was conducted in an algebra class. Forty dice were placed in a cup. After shaking the cup, the dice were rolled on the floor and all the fours were removed. The remaining dice were returned to the cup and the process was repeated until only one die remained. The students used the data to write a function where x represented the roll numbers, 0, 1, 2… and $f(x)$ represented the number of dice remaining. Which function below is the most realistic for modeling the data found?

    a. $f(x) = -3x + 35$ b. $f(x) = -2.9x + 40$ c. $f(x) = .14x^2 - 4.5x + 37$

    d. $f(x) = 39(.85)^x$ e. $f(x) = 40\ (1.16)^x$

30. A three-digit number M can be expressed as a product of prime factors a, b, and $(10a + b)$. Given that $1 \leq a < b < 9$, which of the following is a possible value of M?

    a. 250 b. 378 c. 525 d. 1995 e. none of these

31. Mrs. Lawrence has a yard 30 meters by 36 meters in which a walk of uniform width borders a garden measuring 720 square meters. How wide is the walk?

    a. 30 m b. 12 m c. 7 m d. 3 m e. none of these

32. A die is constructed so that when it is rolled each of the three even outcomes 2, 4, and 6 is twice as likely to come up as each of the odd outcomes 1, 3, and 5. What is the probability that 6 comes up when the die is rolled?

    a. 1 in 6 b. 1 in 9 c. 1 in 2 d. 1 in 3 e. 2 in 9

33. Let f be the function defined on the counting numbers 1, 2, 3, … so that $f(x) = x + 1$ if x is odd and $f(x) = x - 1$ if x is even. Then $f\left((x^2 + 1)^2 + (x^2 - 1)^2\right) =$

a. $2x^4 - 1$ b. $2x^4$ c. $2x^4 + 1$ d. $2x^4 + 2$ e. $2x^4 + 3$

Consider the following for questions 34 and 35.

The graph of a relation is

— symmetric with respect to the x-axis provided that whenever (a, b) is a point on the graph, so is (a, –b);
— symmetric with respect to the y-axis provided that whenever (a, b) is a point on the graph, so is (–a, b);
— symmetric with respect to the origin provided that whenever (a, b) is a point on the graph, so is (–a, –b);
— symmetric with respect to the line y = x, provided that whenever (a, b) is a point on the graph, so is (b, a)

34. The graph of the relation $x^4 + y^3 = 1$ is symmetric with respect to

a. the x-axis b. the y-axis c. the origin d. the line $y = x$
e. both the x-axis and the y-axis.

35. Suppose R is a relation whose graph is symmetric to both the x-axis and the y-axis, and that the point (1, 2) is on the graph of R. Which one of the following points is NOT necessarily on the graph of R?

a. (–1, 2) b. (1, –2) c. (–1, –2) d. (2, 1)
e. All of these points are on the graph of R

36. Suppose the function f is defined so that $f(x) = \begin{cases} 3x & \text{if } x \le 1 \\ (x-1)^2 & \text{if } x > 1 \end{cases}$.

If a is negative then $f(1 - a) =$

a. $(2 - a)^2$ b. a^2 c. $3 - 3a$ d. $3a$ e. $(a - 1)^2$

37. Suppose $a \neq 0$ and $b \neq 0$. An equation for the line passing through $(b, 0)$ that is perpendicular to the line containing both $(a, 0)$ and $(0, b)$ is

a. $y = -\frac{a}{b}x + b$ b. $y = a(x - b)$ c. $y = b\left(1 - \frac{x}{a}\right)$ d. $y = a\left(1 - \frac{x}{b}\right)$

e. $y = a\left(\frac{x}{b} - 1\right)$

38. Suppose $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 1$. Then for any real number x

a. $g(f(x)) = x$ b. $g(f(|x| + 1)) = |x| + 1$ c. $g(f(|x|)) = |x|$
d. $g(f(x)) = |x|$ e. $g(f(|x|)) = x$

39. The hot water faucet in my bathtub can fill the tub in H minutes, the cold water faucet in C minutes. If I first turn on the hot water faucet, and then, a minute later, turn on the cold water faucet as well, how many minutes will it take to fill the tub?

a. $\frac{H+(C-1)}{2}$ b. $\frac{H(C+1)}{C+H}$ c. $\frac{HC}{C+H}$ d. $\frac{HC}{C+H}-1$ e. $\frac{H(C-1)}{C+H}$

40. My grade in a certain math class is determined by three tests. My highest test score will count for 50% of my grade and my lowest for 20%; the "middle" test score will count for 30%. I must have a weighted average of 60 in order to pass the class. If I have already earned scores of 70 on my first two tests, what is the lowest score I can earn on the third test for which I will still pass the class?

a. 40 b. 20 c. 37 d. 70 e. 50

1999 Algebra I State Finals _ Answer Key

1. A	11. C	21. E	31. D
2. C	12. C	22. D	32. E
3. E	13. B	23. C	33. C
4. E	14. E	24. D	34. B
5. E	15. A	25. B	35. D
6. A	16. E	26. E	36. B
7. D	17. C	27. C	37. E
8. D	18. C	28. E	38. B
9. B	19. C	29. D	39. B
10. D	20. B	30. E	40. B