Analytical Geometry:

Solutions:

1. The vertex of the graph of the parabola $y = 3x^2 + bx + c$ is at (-2,-7); determine the value of *b*-*c*; **NCALGII2002:33**
2. The distance between the centers of the circles $x^2 - 8x + y^2 - 28y + 112 = 0$ and $x^6 + 6x + y^2 + 20y - 12 = 0$ is: FUR2003JR11
3. Through which quadrants does the circle $x^2 + 4x + y^2 - 6y + 1 = 0$ pass? FUR2003JR13
4. If a parabola has vertex (0,0) and focus (1,0), the equation is: FUR2003JR!5
5. If Q is the point on the circle $x^2 - 10x + y^2 + 6y + 29 = 0$ which is furthest from the point $P(-1,-6)$, then the distance from P to Q is: FUR2003SR2.

   $x^2 - 10x + y^2 + 6y + 29 = 0 \Leftrightarrow x^2 - 10x + 25 + y^2 + 6y + 9 = -29 + 25 + 9 = 5$
   $\Leftrightarrow (x-5)^2 + (y+3)^2 = 5$

   So this is a circle with center (5,-3) and radius $\sqrt{5}$. The distance from P to Q includes the distance from P to the center and then an additional radius. So this distance is $\sqrt{(5-(-1))^2 + (-3-(-6))^2} + \sqrt{5} = \sqrt{6^2 + 3^2} + \sqrt{5} = \sqrt{45} + \sqrt{5}$

6. The parabola $y = ax^2 + bx + c$ intersects the *y*-axis in the point (0,8) and intersects the *x*-axis in the single point (2,0). How many pairs of integers (m,n) with $-2000 \le m \le 2000$ lie on the parabola? NCSMC2000INT4

   The parabola must be of the form
   $y = a(x-2)^2 \Rightarrow 8 = a(0-2)^2 \Rightarrow a = 2 \Rightarrow y = 2(x-2)^2$. Since the y-values grow much more quickly than the x-values, we need to find all possible points whose y-values are less than or equal to 2000. So
   $2(x-2)^2 \le 2000 \Rightarrow (x-2)^2 \le 1000 \Rightarrow -\sqrt{1000} \le x-2 \le \sqrt{1000}$. But since we want integer coordinates, this means that $-31 \le x-2 \le 31 \Rightarrow -29 \le x \le 33$, so there are 63 such lattice points.

7. Let f(x) be a quadratic polynomial such that f(3) = 15 and f(-3) = -9. Find the coefficient of x in f(x). SMC2002MC1

   We know that $15 = a(3^2) + b(3) + c$ and $-9 = a(-3)^2 + b(-3) + c$, and since we are looking for b, subtract these to get $24 = -6b \Rightarrow b = -4$.

8. The center C and the vertices (V) of the ellipse $4x^2 + 9y^2 - 16x - 54y + 61 = 0$ are: NCALGII2000#9

$$4x^2+9y^2-16x-54y+61=0 \Leftrightarrow 4\left(x^2-4x+4\right)+9\left(y^2-6y+9\right)=-61+16+81=36$$

$$\Leftrightarrow 4\left(x-2\right)^2+9\left(y-3\right)^2=36 \Leftrightarrow \frac{\left(x-2\right)^2}{9}+\frac{\left(y-3\right)^2}{4}=1$$

so the center of this ellipse is (2,3) and vertices (end points of the major axis) are (5,3) and (-1,3).

9. The quadratic equation $y=ax^2+bx+c$ is known to pass through the points (0,5), (2,11) and (-2,15). Find the sum of a and b. NCALGII2000#17

   We know that $11=4a+2b+c, 5=c, 15=4a-2b+c$, so $4a+2b=6, 4a-2b=10 \Rightarrow 8a=16 \Rightarrow a=2$, so $b=-1$ and the sum $a+b=1$.

10. Write an equation for the hyperbola with horizontal transverse axis and asymptotes $y=\pm {}^{4}\!/\!{}_{3}\,x$. NCALGII2000#25

    $$\frac{x^2}{9}-\frac{y^2}{16}=1$$

11. A parabola of the form $y=x^2+bx+c$ contains the points (2,3) and (4,3), Find the value of c. AMC102006.

    We know that $3=4+2b+c, 3=16+4b+c$, so $2b+c=-1, 4b+c=-13 \Rightarrow 2b=-12, b=-6$. This makes $c=11$.