Different Number Bases

Solutions:

1. 3b + 2 = 2c + 1 and 2b + 1 = c + 3 thus c = 2b – 2, so 3b + 2 = 2(2b - 2) + 1 , or 3b + 2 = 4b – 3, so b = 5.
2. $(6b+2)(b+4)=8b^2+8 \Leftrightarrow 6b^2+26b+8=8b^2+8$, so $2b^2-26b=0 \Rightarrow b=0,13$, but since base 0 is meaningless, b = 13.
3. $100a+10b+c-(a+b+c)=126$ and $100c+10b+a-(a+b+c)=225$. Simplify the first equation to get $99a+9b=126 \Leftrightarrow 11a+b=14$, so a must be 1 and b must be 3. Similarly, the second equation simplifies to $99c+9b=225 \Leftrightarrow 11c+b=25$, but since b = 3, c must be 2.
4. Written in base b we have $b+3, 4b+2, b^2+1$ as our numbers, so $(4b+2)-(b+3)=(b^2+1)-(4b+2)$. Simply this to $b^2-7b=0 \Rightarrow b=0,7$, so b must be 7.
5. We have $r=\frac{1}{b}+\frac{1}{b^2}+\frac{1}{b^3}+\cdots$ and $r=2\cdot\left(\frac{1}{2b}\right)+b\left(\frac{1}{4b^2}\right)=\frac{1}{b}+\frac{1}{4b}=\frac{5}{4b}$. The first sum is a geometric series with ratio $\frac{1}{b}, b>1$, so it converges to $r=\frac{1/b}{1-1/b}=\frac{1}{b-1}$, so we have $\frac{1}{b-1}=\frac{5}{4b} \Leftrightarrow 4b=5b-5 \Rightarrow b=5$.