

ALGEBRA II
State Mathematics Contest Finals
May 6, 2004

1. The number of integral values of x that satisfy the inequality $8 < |3x + 4| < 32$ is
 - a. 7
 - b. 8
 - c. 15
 - d. 17
 - e. 22

2. If $1 - \frac{4}{x} + \frac{4}{x^2} = 0$, then $\frac{2}{x}$ equals
 - a. -1
 - b. 1
 - c. 2
 - d. -1 or 2
 - e. -1 or -2

3. For all non-zero numbers x and y such that $x = \frac{1}{y}$, $\left(x - \frac{1}{x}\right)\left(y + \frac{1}{y}\right)$ equals
 - a. $2x^2$
 - b. $2y^2$
 - c. $x^2 + y^2$
 - d. $x^2 - y^2$
 - e. $y^2 - x^2$

4. $|b - a| = |b| - |a|$ is true
 - a. For all real values of a and b .
 - b. If a and b are positive reals.
 - c. For no real values of a and b
 - d. If $b = 0$.
 - e. If $a = 0$ or $a = b$.

5. The product of the values of a , b , and c , that require the graph of $y = ax^2 + bx + c$ to pass through the points $(0, -3)$, $(1, 6)$ and $(2, 9)$ is
 - a. 108
 - b. 56
 - c. 12
 - d. -6
 - e. none of these

6. If Δ is defined by the equation $x\Delta y = x + xy + y$ for all real numbers x and y , what is the value of z if $8\Delta z = 3$?
- 5
 - $-\frac{5}{9}$
 - $\frac{3}{8}$
 - $\frac{5}{9}$
 - 5
7. Assume x is a positive integer. Which of the following cannot be the average of the five numbers 234, 256, 273, 281, and $218x$?
- 218
 - 296
 - 732
 - 1822
 - 2694
8. Mr. Goebel runs y yards in s seconds. What would his rate be, in yards per second, if he ran twice as far in 10 more seconds?
- $\frac{2+y}{10+s}$
 - $\frac{y}{2(s+10)}$
 - $\frac{2y}{10s}$
 - $\frac{2y}{s+10}$
 - $\frac{2y}{s-10}$
9. The collecting of baseball cards has become a popular hobby. One baseball card shows a photograph of Mark McGwire when he was a member of the Olympic USA Baseball Team in 1984. In 1987, the value of the card was \$8 and in 1997, its value was \$20. Assuming the value of the card has grown exponentially with $V_0 = 8$ and there is no change in growth rate, after how long will the value of the card be \$2000?
- 56 years
 - 60 years
 - 64 years
 - 68 years
 - 70 years

10. A triangle has one side of length a and two sides of length b . The area of the triangle is

a. $\frac{a}{4}\sqrt{4b^2 - a^2}$

b. $\frac{a}{2}\sqrt{4b^2 - a^2}$

c. $ab\sqrt{\frac{3}{4}}$

d. $\frac{b}{4}\sqrt{4a^2 - b^2}$

e. $\frac{b}{2}\sqrt{4a^2 - b^2}$

11. Let x be a two digit number such that 0 is not a digit of x and the digits of x are distinct. If the digits of x are reversed to form a new two digit number y , the sum $x + y$ must be divisible by

a. 2

b. 3

c. 5

d. 7

e. 11

12. Determine the value of m such that the three lines $y = 2x + 1$, $x = 3$, and $y = mx + 3$ are concurrent.

a. -1

b. $-\frac{3}{4}$

c. $\frac{3}{4}$

d. 1

e. $\frac{4}{3}$

13. For 2×2 matrices A , B and C , and scalars k and l , which of the following statements is not true?

a. $(A + B)(A - B) = A^2 - B^2$, where $A^2 = AA$ and $B^2 = BB$

b. $k(A + B) = kA + kB$

c. $A(BC) = (AB)C$

d. $A(B + C) = AB + AC$

e. $(k + l)A = kA + lA$

14. If $a + bi = \frac{3-i}{1+i}$, then

- a. $a = 2, b = -2$
- b. $a = 3, b = 1$
- c. $a = -3, b = 1$
- d. $a = 1, b = -2$
- e. $a = 4, b = 4$

15. Suppose the weight of a culture of bacteria doubles daily. If the weight of the culture at the end of k days is M grams, its weight at the end of $k-3$ days is

- a. $\frac{M}{3}$
- b. $\frac{M}{8}$
- c. $8M$
- d. $\frac{M}{9}$
- e. none of these

16. Which of the following statements is true about the solutions of the equation $|x^2 - 5x| = 6$?

- a. The equation has two solutions, both greater than 5.
- b. The equation has two solutions, one positive and one negative.
- c. The equation has three solutions whose sum is 11.
- d. The equation has four solutions whose sum is 10.
- e. None of the above statements is true.

17. A 25 foot tall ladder is placed along the vertical wall of a house. The foot of the ladder is 20 feet from the bottom of the house. If the top of the ladder slips 8 feet, then the foot of the ladder will slide how many feet?

- a. 3 ft.
- b. 5 ft.
- c. 8 ft.
- d. 4 ft.
- e. 7 ft.

18. Mary typed a six-digit number, but the two 1's she typed did not show. What appeared instead was 2002. How many different six-digit numbers could she have typed?

- a. 5
- b. 8
- c. 10
- d. 15
- e. 18

19. The perimeter of a rectangle is 100 cm, and its diagonal has length x . Express the area of the rectangle as a function of x .

a. $\frac{(50-x)^2}{2} \text{cm}^2$

b. $\frac{(50+x)^2}{2} \text{cm}^2$

c. $\frac{2500-x^2}{2} \text{cm}^2$

d. $(2x^2 - 2500) \text{cm}^2$

e. $(2500 + x^2) \text{cm}^2$

20. The difference between the squares of two consecutive odd integers is 128. What is the product of the two integers?

a. 899

b. 1023

c. 1155

d. 783

e. 483

21. If S and T are sets such that S has 2 more elements than T and 96 more subsets than T , how many elements are in S ?

a. 3

b. 5

c. 7

d. 9

e. 11

22. If $(-1,5)$ is a point of the graph of $y = f(x)$, then the graph of $y = f(x-3)$ contains the point $(2,c)$ where c equals

a. 2

b. 3

c. 4

d. 5

e. 6

23. A function f is even if for each x in the domain of f , $f(x) = f(-x)$. A function f is odd if for each x in the domain of f , $f(-x) = -f(x)$. Which of the following statement(s) is (are) true?

- I. The product of two odd functions is odd
- II. The sum of two even functions is even.
- III. The product of an even function and an odd function is odd.
- IV. If f is any function and the function F is defined by $F(x) = \frac{f(x) + f(-x)}{2}$, then F is even.

- a. All statements are true.
- b. Only I, II and III are true.
- c. Only II, III and IV are true.
- d. Only II and III are true.
- e. Only III and IV are true.

24. The volume of wood V in a tree varies jointly as the height h and the square of the girth g (girth is the distance around). If the volume of a redwood tree is 216 m^3 when the height is 30 m and the girth is 1.5 m, what is the height of a tree whose volume is 960 m^3 and girth is 2m?

- a. 14.8 m.
- b. 45 m.
- c. 50 m.
- d. 75 m.
- e. 95 m.

25. Compute the area of the region enclosed by the graph $|2x - 10| + |5y - 10| = 20$.

- a. 80
- b. 85
- c. 86
- d. 90
- e. 96

26. Given $a_3 = 5$, $a_5 = 8$ and $a_n + a_{n+1} + a_{n+2} = 7$ for all positive integers, n , compute a_{2001} .

- a. 5
- b. 6
- c. 8
- d. -6
- e. -8

27. Compute N so that $\frac{N}{x-5} + \frac{3}{x+4} = \frac{10x+13}{x^2-x-20}$ for all $x > 2000$.
- 4
 - 7
 - 10
 - 13
 - 16
28. Compute the maximum value of $x + y$ for all positive integer pairs (x, y) that satisfy $15x + 55y = 2000$.
- 104
 - 120
 - 128
 - 172
 - 196
29. Solve $(a + bi)^2 = 5 + 12i$ for a and b . The number of solutions is
- 0
 - 1
 - 2
 - 3
 - 4
30. Let b be a positive integer and consider $f(x) = 2x^2 + bx + 10$. As b increases, how does the graph of $f(x)$ change?
- The vertex changes with the x -coordinate decreasing at a slower rate than the y -coordinate.
 - The x -coordinate of the vertex increases by 1 whenever the y -coordinate of the vertex decreases by 6.
 - The vertex changes with the x -coordinate decreasing while the y -coordinate remains constant.
 - The x -coordinate of the vertex decreases by 1 whenever the y -coordinate of the vertex decreases by 4.
 - None of these.
31. The price of a computer over a 25 month period is described by $f(x) = 1050 - \sqrt[3]{9x^5 + 5x + 503}$. If the model continues to be accurate beyond the 25 month period, approximately when will the price reach half of its initial price?
- 26 months, 29 days.
 - 27 months, 19 days
 - 27 months, 22 days.
 - 28 months.
 - 28 months, 3 days.

32. A luxury car's value is represented by the equation $y_1 = 50(0.822)^x$. A sports utility vehicle's (SUV) value is represented by the equation $y_2 = 30(0.884)^x$. In both functions, y is the value (in \$1000's) of the automobile after x years. Assuming one of each model is purchased on the same day, how long will it be before the SUV is worth \$1000 more than the luxury car?
- 5.3 years.
 - 6.1 years.
 - 7.4 years.
 - 8.1 years.
 - None of these.
33. A dress that is size x in France is size $s(x)$ in the United States, where $s(x) = x - 32$. A dress that is size x in the United States is size $y(x)$ in Italy, where $y(x) = 2(x + 12)$. Which function $h(x)$ will convert French dress sizes to Italian dress sizes?
- $h(x) = 2(x - 20)$
 - $h(x) = 2(x - 4)$
 - $h(x) = 3x - 8$
 - $h(x) = x + 56$
 - $h(x)$ cannot be determined from the information provided.
34. Suppose that $\log_b 3 = 1.0986$ and $\log_b 5 = 1.6094$. Then the value of $\log_b 75$, accurate to four decimal places, is
- 2.2422
 - 2.8456
 - 3.5362
 - 3.8066
 - 4.3174
35. The Container Company is designing an open top rectangular box, with a square base, that will hold 108 cm^3 . Estimate the minimum surface area for the box.
- 120 cm^2
 - 108 cm^2
 - 102 cm^2
 - 96 cm^2
 - 92 cm^2

36. Dovetail Carpentry Shop makes bookcases and desks. Each bookcase requires 5 hours of woodworking and 4 hours of finishing. Each desk requires 10 hours of woodworking and 3 hours of finishing. Each month the shop has 600 hours of labor available for woodworking and 240 hours for finishing. The profit on each bookcase is \$40 and on each desk is \$75. How many of each product should be made in order to maximize profit?
- 48 bookcases, 24 desks.
 - 60 bookcases, 0 desks.
 - 57 bookcases, 4 desks.
 - 12 bookcases, 54 desks.
 - None of these.
37. A rectangular field is 300 feet wide and 400 feet long. Random sampling indicates that there are, on the average, three ants per square inch throughout the field. Of the following, the number that most closely approximates the number of ants in the field is
- 500 thousand.
 - 5 million.
 - 50 million.
 - 500 million.
 - 5 billion.
38. For how many three-element sets of positive integers $\{a, b, c\}$ is it true that $a \times b \times c = 2310$?
- 32
 - 36
 - 40
 - 43
 - 45
39. Let f be a linear function with the properties that $f(1) \leq f(2)$, $f(3) \geq f(4)$ and $f(5) = 5$. Which of the following statements is true?
- $f(0) < 0$.
 - $f(0) = 0$.
 - $f(1) < f(0) < f(-1)$
 - $f(0) = 5$
 - $f(0) > 5$
40. The number of distinct real values x which satisfy the equation $(x^2 - 5x + 5)^{x^2 - 9x + 20} = 1$ is
- 0
 - 2
 - 3
 - 5
 - 6