

State Mathematics Finals: Algebra I
May 4, 2006

1. Solve the following for x : $2x + c = 6$.

- a) $3 - c$ b) $\frac{3-c}{2}$ c) $6 - 2c$ d) $3 - \frac{c}{2}$ e) $6 - \frac{c}{2}$

2. Abe earns \$5 an hour helping his neighbor do yard work. Abe is trying to save up for a new iPod which will cost him \$300 + 7% tax. If he has already saved \$111, how many hours does he have to work in order to save enough money for the iPod?

- a) 37.8 hrs. b) 39.2 hrs c) 42 hrs d) 64.4 hrs. e) 210 hrs.

3. Simplify $\frac{2}{A+1} - \frac{1}{A-1} + \frac{2A}{A^2-1}$

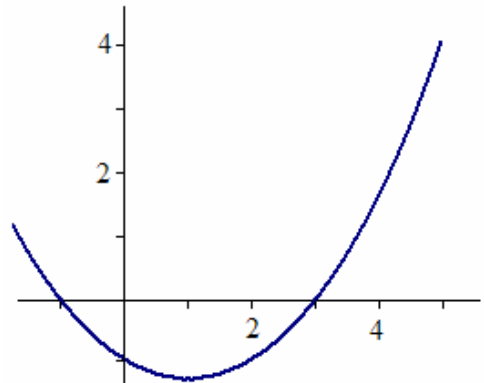
- a) $\frac{3}{A+1}$ b) $\frac{2A+3}{A^2-1}$ c) $\frac{5}{A-1}$ d) $\frac{3A-1}{A^2-1}$ e) $\frac{2}{A-1}$

4. Find the next number in the following number sequence: 0, 1, 5, 14, 30, 55, ...

- a) 71 b) 27 c) 84 d) 89 e) 91

5. Which function is depicted in the figure on the right?

- a) $f(x) = (x+1)(x-3) + 2$
b) $f(x) = (x-1)^2 - 4$
c) $f(x) = 0.3x^2 - 0.7x + 1.3$
d) $f(x) = \frac{1}{3}(x-1)(x+3)$
e) $f(x) = \frac{1}{3}(x+1)(x-3)$



6. If $x + y = 17$ and $x^2 + y^2 = 167$ then what is the value of xy ?
- a) $17 + 4\sqrt{114}$ b) 61 c) $48 + \sqrt{167}$ d) 122 e) $17 - \sqrt{167}$
7. Find the smallest natural number, x , for which 3 divides x , 5 divides $(x+1)$ and 7 divides $(x+2)$.
- a) 159 b) 105 c) 60 d) 54 e) 14
8. Solve for k given $\frac{16^k + 16}{16} = 9$
- a) 2 b) $\frac{4}{7}$ c) $\frac{7}{4}$ d) $\frac{9}{5}$ e) 8
9. Find the sum: $3 + 7 + 11 + 15 + 19 + 23 + \dots + 399$.
- a) 20,100 b) 79,800 c) 19,950 d) 39,900 e) none of these
10. For what value of c will the following two equations NOT have a solution?
- $$12x + cy = 64 \quad \text{and} \quad 5x - 2y = 10$$
- a) 2.4 b) -2.4 c) -2.5 d) -5 e) -4.8
11. At 4:00 the angle between the minute hand and the hour hand on a standard analog clock is 120° . How much time will elapse until the two hands will again form an angle of 120° ?
- a) 40 min. b) 42 min. c) $43\frac{7}{11}$ min d) $47\frac{7}{12}$ min e) 60 min

12. A thumbtack has probability of 0.7 that it lands with the point sticking up. If 4 thumbtacks are dropped what is the probability that exactly 3 of them will have the point sticking up?
- a) 0.4116 b) 0.5884 c) 0.1029 d) 0.8971 e) none of these
13. If $\sqrt{x} = \sqrt[3]{9}$, then what does x^2 equal to?
- a) $\sqrt[3]{81}$ b) 3 c) $9\sqrt[3]{9}$ d) $\sqrt[3]{9}$ e) $9\sqrt{3}$
14. Beth left town at 1 PM and was driving at 60 mph, and Carl left 15 minutes later and was driving at 70 mph. How far out of town were they when Carl reached Beth?
- a) 105 mi. b) 90 mi. c) 25 mi. d) 12.5 mi. e) none of these
15. Consider the parabola, $y = 3x - x^2$. Find the values of “k” so that the line, $y = kx + 3$ will intersect the parabola exactly once?
- a) 0 b) $3 \pm \sqrt{12}$ c) $3 \pm \sqrt{5}$ d) -1, 7 e) none of these
16. Each letter in the expression below represents a unique digit, and different letters never have the same values. Find the digit that corresponds to the letter “X”.
- $$\text{AMMA} \times \text{XA} = \text{AAAAAA}$$
- a) 2 b) 3 c) 5 d) 7 e) 9
17. Today Debbie earns twice as much per hour as she earned a year ago, and she works 12 hours per week more, resulting in a total weekly income that is 3 times higher than it was last year. How many hours does she presently work per week?
- a) 12 b) 24 c) 32 d) 36 e) none of these

18. Define the operation, “★”, on real numbers as $a \star b = \frac{a+b}{2}$,

Evaluate the following $((x-2) \star (x+4)) \star (x+7)$.

- a) $x+3$ b) $x+3.5$ c) $3x+9$ d) $3x+4$ e) $x+4$

19. Define the operation, “★”, as in problem 29 and solve for x in terms of c .

$$((x-c) \star x) \star (x+c) = x \star 2c.$$

- a) $\frac{c}{4}$ b) $\frac{3c}{4}$ c) $\frac{5c}{4}$ d) $\frac{7c}{4}$ e) none of these

20. Fred purchased several digital cameras for a total of \$4,284 and sold each with a 50% mark up on e-bay. However each transaction took about \$21 to process, and he needed to sell 12 cameras in order to break even. What were his profits after he sold all the cameras?

- a) \$1,785 b) \$1,890 c) \$2,142 d) \$6,069 e) none of these

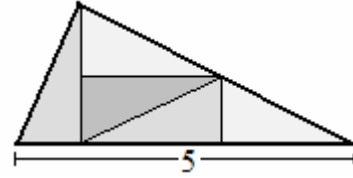
21. Let $\frac{1}{x + \frac{1}{x}} = p$ Solve for x in terms of p .

- a) $\frac{p}{p+1}$ b) $\frac{1 \pm \sqrt{1-4p^2}}{2p}$ c) $\frac{p \pm \sqrt{p^2-4}}{2}$ d) $\frac{p}{1-p}$ e) none of these

22. Find the solution set of $\frac{x^2-4}{x-4} \leq 0$.

- a) $\{x | (x \leq 2) \cup (x > 4)\}$ b) $\{x | (x \leq -2) \cup (2 \leq x < 4)\}$ c) $\{x | (x \leq 2) \cup (x \geq 4)\}$
d) $\{x | (-2 \leq x \leq 2) \cup (x > 4)\}$ e) none of these

23. A right triangle is partitioned into five congruent triangles as shown in the figure to your right. If the hypotenuse has length 5 find the length of the shorter leg.



- a) $\sqrt{5}$ b) $\frac{5-\sqrt{5}}{2}$ c) 3 d) $\frac{1+\sqrt{5}}{2}$ e) $5-\sqrt{5}$
24. A triangular number is a number of the form $\frac{1}{2}n(n+1)$. How many triangular numbers are greater than 100 but less than 200?
- a) 6 b) 7 c) 8 d) 9 e) 10
25. A set of tennis balls are stacked in a triangular pyramid, the base of which forms an equilateral triangle composed of 66 balls. The next layer reduces each side by one ball, until you reach the top. How many balls are in the pyramid?
- a) 220 b) 254 c) 286 d) 364 e) 2,211
26. Grace lives in a town with roads that are laid out as a rectangular grid. Her friend lives 8 blocks away, 2 South and 6 East. How many different paths (each 8 blocks long) can Debbie take to reach her friend's house?
- a) 20 b) 22 c) 25 d) 27 e) 28
27. Simplify $\frac{\sqrt{5}}{\sqrt{x}-\sqrt{5}} + \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5}}$
- a) $\frac{x+4}{x-1}$ b) $\frac{x+20}{x-5}$ c) $\frac{x-20}{x+5}$ d) $\frac{1}{2}$ e) $\frac{x+5}{x-5}$
28. Find the sum of the two solutions to the equation: $4^{2x+1} + 8 = 33(4^x)$.
- a) -1 b) -0.5 c) 0.5 d) 1 e) 1.5

29. Harold and Ida visit their grandparents periodically, Harold every 10 days and Ida every 28 days. They both were with their grandparents at New Year's Eve. How long will it be until the next time they are all together again?
- a) 365 days b) 280 days c) 210 days d) 140 days e) 70 days
30. Given the base 4 numbering system with digits 0, 1, 2 and 3, what is $(231 + 122) \times 21$?
- a) 21,333 b) 20,013 c) 22,203 d) 21,143 e) none of these
31. John's grades in Algebra are: homework grade – 65, quiz grade – 82, project grade – 93. If homework is 20% of the grade, quizzes are 30%, projects 20%, and the remaining grade is based on the final, what grade does John need on the final exam to get an 85 average?
- a) 90 b) 92 c) 94 d) 96 e) 100
32. Let $M(n)$ be the function defined on the natural numbers that multiplies all the digits of n . Define $M^*(n)$ to be function that repeats the process on n until you are down to a single digit number.
 Example: $M(99) = 81$ and $M(81) = 8$ thus $M^*(99) = 8$.
 What proportion of two digit numbers have $M^*(n) = 6$?
- a) 1/10 b) 1/9 c) 11/90 d) 2/15 e) 13/90
33. If, $\frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots = s$ then what is the value of the sum $\frac{1}{4} + \frac{1}{16} + \frac{1}{36} + \frac{1}{64} + \dots$.
- a) $\frac{s}{2}$ b) $\frac{s}{4}$ c) $\frac{s-1}{4}$ d) $\frac{s+1}{4}$ e) none of these
34. Find the slope of the line connecting the points of intersections of the functions,
 $f(x) = x^2 - 2x + 2$ and $g(x) = 2(x^2 - 3x - 5)$.
- a) 2 b) $\frac{13}{6}$ c) $\frac{1}{2}$ d) $\frac{8}{3}$ e) none of these

35. In this “Mini-Sudoku” puzzle you fill in the grid so that every row, every column, and every 2×2 box contains the digits 1 through 4. Determine the product of the values of the inner four cells.

	2		1
3			
		4	

- a) 6 b) 8 c) 12 d) 16 e) 24

36. Find the distance between the points of intersection of $x^2 + y^2 = 25$ and $x + 2y = 5$.

- a) 4 b) 80 c) $4\sqrt{5}$ d) $5\sqrt{2}$ e) none of these

37. Find the values a and b such that $x^2 - 2a(x + 3) = b(x + 3)^2 + 3^2$. What is the value of $b - 2a$?

- a) 3 b) 4 c) 5 d) 7 e) 11

38. If $a + b + c + d = 15$, $a - b + c - d = 3$ and $a + b - c - d = 7$ determine $b + 2c + 3d$.

- a) 25 b) 20 c) 14 d) 10 e) can not be determined

39. A right triangle with a perimeter of length 100 has an area of 400. What is the length of its hypotenuse?

- a) $8\sqrt{41}$ b) 45 c) $29 + \sqrt{41}$ d) $24\sqrt{5}$ e) 42

40. Every year the number of starlings doubles in the spring, but that number is reduced by 15 in the summer through predation by cats. If you start with 20 starlings in the first winter, how many will you have “ t ” years later?

- a) $19 + 2^t$ b) $20 + 2t$ c) $20 \cdot 2^t - 15$ d) $5(3 + 2^t)$ e) $20 + 5t \frac{t+1}{2}$

Algebra I: Answer Sheet

May 4, 2006

- | | | | |
|-----|---|-----|---|
| 1. | D | 21. | B |
| 2. | C | 22. | B |
| 3. | A | 23. | A |
| 4. | E | 24. | A |
| 5. | E | 25. | C |
| 6. | B | 26. | E |
| 7. | D | 27. | E |
| 8. | C | 28. | C |
| 9. | A | 29. | D |
| 10. | E | 30. | A |
| 11. | C | 31. | D |
| 12. | A | 32. | E |
| 13. | C | 33. | D |
| 14. | A | 34. | A |
| 15. | B | 35. | B |
| 16. | E | 36. | C |
| 17. | D | 37. | D |
| 18. | E | 38. | C |
| 19. | E | 39. | E |
| 20. | A | 40. | D |

Best of three: 10, 20, 30

Sudden Death: 5, 15, 25, 35, 40