

Algebra II
State Mathematics Contest Finals
May 1, 2008

1. Find the y value of the intersection of $y = 2x + 7$ and $4x + 3y = 4$.
a) -1.7 b) 2.2 c) 3.5 d) 3.6 e) 5.25

2. For what value of b will $|2x - 1| = x + b$ have a unique solution?
a) -0.5 b) 0 c) 0.5 d) 1.5 e) 2

3. Let operator " \diamond " be defined on the set $\{p, r, s, t, u\}$, and assume " \diamond " is associative and commutative. If $p \diamond r = s$, $p \diamond p = t$ and $r \diamond t = u$. Find $p \diamond s$.
a) p b) r c) s d) t e) u

4. What is the sum of all the odd numbers from 1 to 2008?
a) $1,008,016$ b) $1,010,025$ c) $1,008,518$ d) $2,015,028$ e) $2,017,036$

5. In a two way election between HC and BO, 55% of the voters were women and of those 60% voted for HC. Only 62% of the men voted for BO. What percentage of the vote did HC get?
a) 49% b) 50.1% c) 52.5% d) 53.5% e) 61%

6. Solve $9^x - 3^x = 12$.
a) $\log_4 3$ b) $\log_3 12$ c) $\log_3 4$ d) $\sqrt{11} - 2$ e) $\log_6 12$

7. Which of the following expressions, if any, is different from the other 3? ($i = \sqrt{-1}$, $c \in \text{Reals}$)

a) $\frac{1}{2} - \frac{i+c}{i-1}$ b) $\frac{c}{2} + \frac{i+ci}{2}$ c) $\frac{i}{2} + \frac{c}{i+1}$ d) $\frac{-1+(2c+1) \cdot i}{2(1+i)}$ e) All are the same.

8. The location of a robot on the Cartesian plane at time t is given by (x, y) , where $x = 1.6t + 2$ and $y = t^2 - 2t$. What is the distance between the robot's location at $t = 0$ and $t = 5$?

a) 15 units b) 17 units c) $5\sqrt{13}$ units d) 23 units e) 25 units

9. Jeremy has a bicycle repair shop and needs to know how much to charge for labor. If he charges too much he will lose customers; if he charges too little he won't make much money. At the present, he charges \$40 per hour and has 15 hours of work a week. He knows that for every \$5 increase in the hourly rate his workload drops by 3 hours, and for every \$5 decrease his workload goes up by 3 hours. How much should Jeremy charge per hour to maximize his profits?

a) \$42.50 b) \$40.00 c) \$35.00 d) \$32.50 e) \$30.00

10. If $x - y = 7$ and $x^2 + 3xy - 4y^2 = 14$, find the value of $x + y$.

a) 0 b) 2 c) 5 d) $10\frac{1}{3}$ e) $14\frac{1}{7}$

11. A class with 7 women and 5 men has 4 students chosen at random. If all have an equal chance of being picked, what is the chance that the group will have 2 men and 2 women?

a) $\frac{1}{3}$ b) $\frac{14}{33}$ c) $\frac{7}{33}$ d) $\frac{7}{99}$ e) $\frac{1225}{3456}$

12. For what value of m will the triangle formed by the lines $y = 4$, $x = 0$ and $y = m \cdot x - 2$ have an area of 42?

a) $\frac{1}{2}$ b) $\frac{11}{42}$ c) $\frac{4}{14}$ d) $\frac{4}{21}$ e) $\frac{3}{7}$

13. Let $n\downarrow$ denote the largest prime number less than n and $n\uparrow$ denote the smallest prime number greater than n . Find the value of $((10 + 20\uparrow)\downarrow + 30)\uparrow$

- a) 57 b) 59 c) 60 d) 61 e) 67

14. Three standard dice are rolled, and someone tells you that the total of the top faces is 13. What is the probability that any one die has the number five on the top face?

- a) $\frac{1}{2}$ b) $\frac{4}{7}$ c) $\frac{11}{21}$ d) $\frac{91}{216}$ e) $\frac{3}{216}$

15. Let $\lfloor x \rfloor$ denote the largest integer less than or equal to x , and $\lceil x \rceil$ denote the smallest integer greater than or equal to x . If x is NOT an integer which of the following is FALSE?

- a) $\lceil x \rceil - \lfloor x \rfloor = 1$ b) $\lfloor x + 2 \rfloor = \lfloor x \rfloor + 2$ c) $\lceil 2x \rceil \leq 2 \lceil x \rceil$
d) $\lceil -x \rceil = -\lfloor x \rfloor$ e) $\lfloor \lceil x \rceil + x \rfloor = \lceil x + \lfloor x \rfloor \rceil$

16. Let $g(x) = x^2 + b \cdot x + c$ and $g(2) = -6$. Determine $g(5)$.

- a) -15 b) $2c - 9$ c) $-1.5c$ d) $2.5c - 10$ e) $-4c$

17. The expression $\sqrt{5 + \sqrt{5 + \sqrt{5 + \sqrt{5 + \dots}}}}$ reduces to which of the following number?

- a) $\frac{1 + \sqrt{21}}{2}$ b) $\frac{1 + 2\sqrt{5}}{2}$ c) $\frac{5 + \sqrt{5}}{2}$ d) $\frac{5 - \sqrt{5}}{2}$ e) 3

18. Find the area of a triangle whose vertices are $(1, 3)$, $(5, 7)$, and $(9, -1)$.

- a) 18 b) $5\sqrt{17}$ c) $8\sqrt{10}$ d) 24 e) 48

25. Sarah has 22 coins in her purse that total \$10.50 in value. Assuming these coins are dollar coins, quarters, and nickels, and that she has at least one of each kind, how many dollar coins does she have?

- a) 3 b) 5 c) 6 d) 7 e) 8

26. Today is Thursday May 1. What day of the week will May 1, 2018 be?

- a) Saturday or Sunday b) Monday c) Tuesday d) Wednesday e) Thursday or Friday

27. The sum of the repeated decimal fractions, $3.1\overline{45}$ and $4.1\overline{54}$, is:

- a) 7.3 b) $7.\overline{299}$ c) $7.\overline{2991}$ d) $7.\overline{2996086}$ e) $7.\overline{2996087}$

28. A rubber ball is dropped from a height of 4 meters onto a concrete floor. It bounces to 75% of its original height and drops back down, bouncing many times in ever smaller bounces. If it did this forever how much total distance did the ball cover?

- a) 10m b) 24m c) 28m d) 42m e) more than 42m

29. Solve $\sqrt{2-x} + x = 1$. If there are multiple solutions, give the largest one.

- a) 1 b) $\frac{1-\sqrt{5}}{2}$ c) $3-\sqrt{13}$ d) $\frac{1+\sqrt{5}}{2}$ e) No solution exists

30. Find the equation of a line that is tangent to circle $x^2 + 4x + y^2 + 6y = 12$ at point (1, 1).

- a) $3x + 4y = 7$ b) $4x + 6y = 12$ c) $4x - 3y = 1$ d) $3x - 4y = -1$ e) $3x - 2y = 1$

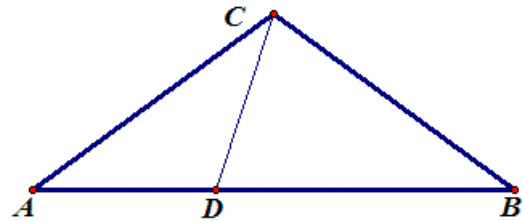
31. Given $f(x) = 0.5x^2 - 2x$, find the slope of the line connecting $(0, f(0))$ to $(6, f(6))$.

- a) 1 b) 1.5 c) 2 d) 2.25 e) 4

32. If $x^2 = x + 1$ then which of the following expressions is equal to x^6 ?

- a) $6x + 4$ b) $8x + 5$ c) $x^5 + 1$ d) $5x + 3$ e) $5x + 8$

33. An isosceles triangle ABC with point D on \overline{AB} , has $AC = BC = BD$ and $AD = DC$. If $AB = 2$, find the length of \overline{CD} .



- a) 1 b) $\sqrt{2}$ c) $\sqrt{5} - 1$ d) $\sqrt{10} - 3$ e) $3 - \sqrt{5}$

34. The perimeter of a right triangle is six times longer than its shortest side. What is the value of the ratio of the longer leg to the shorter one?

- a) $1.08\bar{3}$ b) 1.2 c) 2.4 d) 2.6 e) $2.\bar{6}$

35. The equation $||x + 5| - 3| = c$, where c is a real number has exactly three distinct solutions. Find the value of c .

- a) 0 b) 2 c) 3 d) 5 e) 7

36. If $f(1 - t^{-1}) = (5t + 1)/t$ then $f(t) = ?$

- a) $6 - t$ b) $\frac{6t - 1}{t - 1}$ c) $\frac{t + 1}{4t + 1}$ d) $5 + \frac{1}{t}$ e) $\frac{1 - 4t}{5t + 1}$

37. Abigale is 10 years older than Bertha who is twice as old as Charlotte who is as old Dorothy and Eva combined. If Abigale is 5 times as old as Eva, and Bertha is 23 years older than Dorothy, how old is Charlotte?

- a) 7 b) 15 c) 21 d) 30 e) 40

38. $P = P_0(1 + \alpha)^t$, determines the amount of money, P , in an account that accrues α interest that is compounded annually over a period of t years. P_0 is the amount of money you start with. Which expression calculates the time it takes for your money to triple?

- a) $2/\alpha$ b) $(P - P_0) / \alpha$ c) $\log(3)/\alpha$ d) $3/\log(1 + \alpha)$ e) $\log(3)/\log(1 + \alpha)$

39. Which of the following expression is NOT a factor of $x^6 - 64$.

- a) $x - 2$ b) $x + 2$ c) $x^2 + 2x + 4$ d) $x^2 + 4$ e) $x^2 - 2x + 4$

40. The following is a 5×5 Sudoku puzzle. The digits 1-5 appear exactly once in each row, each column, and each region. Which of the digits should be in the shaded square?

- a) 1 b) 2 c) 3 d) 4 e) 5

