GEOMETRY

STATE FINALS MATHEMATICS CONTEST
May 1, 2008

1. Consider 3 squares *A*, *B*, and *C* where the perimeter of square *A* is $\frac{2}{3}$ the perimeter of square *B*, and the perimeter of square *B* is $\frac{2}{3}$ the perimeter of square *C*. If the area of square *A* is 16, what is the area of square *C*?

   a. 24.
   b. 36.
   c. 64.
   d. 72.
   e. 81.

2. If the circumference of circle I is equal to the diameter of circle II, what is the ratio of the area of circle II to the area of circle I?

   a. $\frac{1}{\pi^2}$.
   b. $\sqrt{\pi}$.
   c. π .
   d. π^2.
   e. $4\pi^2$.

3. If circle *O* has its center at (1,1) and line *L* is tangent to circle *O* at P(4,-4), what is the slope of *L*?

   a. $-\frac{5}{3}$.
   b. $-\frac{3}{5}$.
   c. $\frac{3}{5}$.
   d. 1.
   e. $\frac{5}{3}$.

4. A white cube has a volume of 27. If a red circle of radius 1 is painted on each face of the cube, what is the total area of the surface of the cube that is NOT red?

   a. 6π.
   b. 12π.
   c. $27 - 6\pi$.
   d. $54 - 6\pi$.
   e. $54 - 12\pi$.

5. Suppose that the coordinates of *A* and *D* are (1,4) and (1,10) respectively and that *ABCD* forms a square with the *x* coordinate of *B* greater than 1. If *F* has coordinates (10,0), what is the area of ΔBFC?

   a. 6.
   b. 9.
   c. 10.
   d. 14.
   e. 18.

6. Suppose a rectangle has area 10 and a diagonal length $\sqrt{29}$. What is its perimeter?

   a. 14.
   b. 16.
   c. 18.
   d. 20.
   e. 22.

7. Find the radius of a circle circumscribed about a triangle with sides of lengths 15, 36 and 39.

   a. 15.
   b. 18.
   c. 19.5
   d. 37.5.
   e. 39.

8. Consider the circles with radii $4\sqrt{5}$ and which are tangent to the line $x - 2y = 20$ at the point (6,-7). The sum of the *x* coordinates of the centers of the circles is

   a. 12.
   b. -14.
   c. 3.
   d. -5.
   e. 2.

9. Points *M* and *N* are the midpoints of sides $\overline{PA}$ and $\overline{PB}$ of ΔPAB. As *P* moves along a line that is parallel to side $\overline{AB}$, how many of the four quantities listed below change?

   I. the length of the segment $\overline{MN}$.
   II. the perimeter of ΔPAB
   III. the area of ΔPAB
   IV. the area of trapezoid *ABNM*

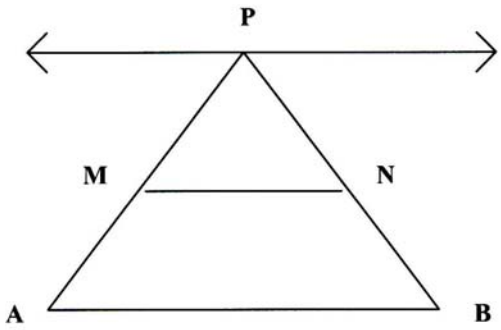
   a. 0.

   b. 1.

   c. 2.

   d. 3.

   e. 4.



10. In rectangle *ABCD*, $AD = 1$, *P* is on $\overline{AB}$, and $\overline{DB}$ and $\overline{DP}$ trisect $\angle ADC$. What is the perimeter of ΔBDP?

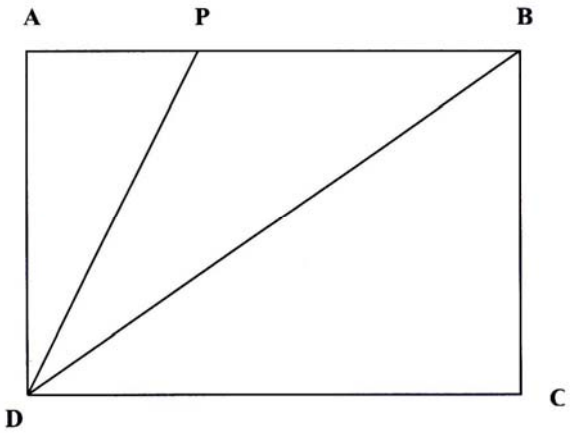
    a. $3 + \frac{\sqrt{3}}{3}$.

    b. $2 + \frac{4\sqrt{3}}{3}$.

    c. $2 + 2\sqrt{2}$.

    d. $\frac{3 + 3\sqrt{5}}{2}$

    e. $2 + \frac{5\sqrt{3}}{3}$



11. Jonathan walks completely around the boundary of a square whose sides are each 5 km long. From any point on his path he can see exactly 1 km horizontally in all directions. What is the area of the region consisting of all points Jonathan can see during his walk, expressed in square kilometers and rounded to the nearest whole number?

    a. 16.
    b. 24.
    c. 27.
    d. 39.
    e. 42.

12. In a regular hexagon, the diagonals are increased by 30 percent. By what percentage will the area of the hexagon increase as a result?

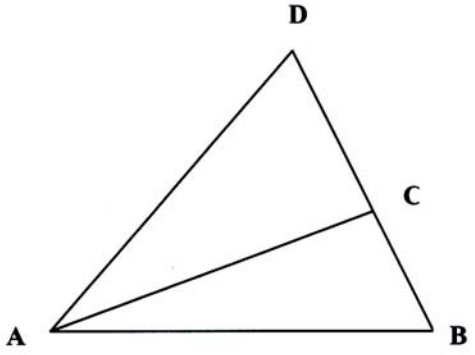
    a. 30.
    b. 40.
    c. 46.
    d. 60.
    e. 69.

13. A right triangle with integer side lengths a, b, and c satisfies $a < b < c$ and $a + c = 49$. What is the area of the right triangle?

    a. 176.
    b. 210.
    c. 224.
    d. 225.
    e. 232.

14. In $\Delta ABC, AB = 20, BC = 7$ and $CA = 15$. $\overline{BC}$ is extended to point D so that ΔDAB is similar to ΔDCA. What is DC? (Figure is not drawn to scale)

    a. 7.
    b. 9.
    c. 11.
    d. 12.
    e. 14.



15. An isosceles right triangle is removed from each corner of a square piece of paper so that a rectangle remains. What is a length of a diagonal of the rectangle if the sum of the areas of the removed pieces is 200?

    a. 10.
    b. 20.
    c. 40.
    d. $10\sqrt{2}$.
    e. $14\sqrt{2}$.

16. A ladder leans against a house with its base 15 feet from the house. When the ladder is pulled 9 feet farther away from the house, the upper end slides down 13 feet. How long is the ladder?

    a. 20.
    b. 22.
    c. 24.
    d. 25.
    e. 28.

17. A piece of construction paper 0.01 mm thick is cut in half, and one piece is placed on the other to make a pile. These are cut in half, and all four pieces are placed in a pile.
    These four are cut in half and placed in a pile and the process continues. After the pieces have been cut and piled for the 10^{th} time, what is the height of the pile in cm?

    a. .5 cm.
    b. .512 cm.
    c. 1 cm.
    d. 1.024 cm.
    e. 2.048 cm.

18. Let C_1 denote a circle that is circumscribed about a square of side *s*. Let C_2 denote a
    circle of radius *s* that is centered at a vertex of the square. The area of the region which is inside C_1 and outside C_2 is:

    a. $\frac{\pi}{\sqrt{2}} \cdot s^2$.
    b. $\frac{\pi}{4} \cdot s^2 - \frac{\pi}{\sqrt{2}} \cdot s^2$
    c. $\frac{\pi}{4} \cdot s^2$.
    d. s

e. $\frac{1}{2} \cdot s^2$.

19. The triangle with vertices *A*, *B* and *C* satisfies the following conditions:

the degree measure of $\angle A$ is 30°,
the degree measure of $\angle C$ is 50°,
and the length of side $\overline{AC}$ is 10 inches.

The area of the triangle is:

a. less than 15 square inches.
b. at least 15 but less than 16 square inches.
c. at least 16 but less than 17 square inches.
d. at least 17 but less than 18 square inches.
e. at least 18 square inches.

20. The line $y = -\frac{3}{4}x + 9$ crosses the *x*-axis at *P* and the *y*-axis at *Q*. Point *T* lies on $\overline{PQ}$ between *P* and *Q* and its coordinates are (*r*,*s*). If $\frac{\text{Area of } \Delta POQ}{\text{Area of } \Delta TOP} = 3$, find the value of $r + s$.

a. 10.
b. 10.5.
c. 11.
d. 11.5.
e. 12.

21. How many triangles with positive area are there whose vertices are points in the *xy*-plane whose coordinates are integers (*x*,*y*) satisfying $1 \le x \le 4$ and $1 \le y \le 4$?

a. 496.
b. 500.
c. 512.
d. 516.
e. 560.

22. A circle of radius 2 has its center at (2,0). A circle of radius 1 has its center at (5,0). A line is tangent to the two circles at points in the first quadrant. Find the *y*-intercept of the line.

    a. $2\sqrt{2}$.
    b. $\frac{\sqrt{2}}{4}$.
    c. 3.
    d. $\frac{8}{3}$.
    e. $1+\sqrt{3}$.

23. A rectangular field is 300 feet wide and 400 feet long. Random sampling indicates that there are, on the average, three ants per square inch throughout the field. Of the following, the number that most closely approximates the number of ants in the field is

    a. 500 thousand.
    b. 5 million.
    c. 50 million.
    d. 500 million.
    e. 5 billion.

24. A rectangular box with no top has a base that is a 2 ft by 3 ft rectangle and a volume of 6 cubic ft. A fly crawls from one corner at the top of the box to the diagonally opposite corner at the top of the box. What is the minimum distance the fly can crawl, provided the fly's path takes it into the base of the box?

    a. $\sqrt{17}$.
    b. 5.
    c. $\sqrt{29}$.
    d. $2+\sqrt{13}$.
    e. 7.

25. Two ladders, one of which is three times as long as the other, rest on the floor and reach the same vertical height on the wall. The shorter ladder makes an angle of 60° with the floor. What angle does the longer ladder make with the floor? Round to the nearest degree.

    a. 16°.
    b. 17°.
    c. 18°.
    d. 20°.
    e. 21°.

26. Suppose that *ABCD* is a parallelogram, with the vertices listed in clockwise order, and that *E* is the midpoint of $\overline{BC}$ and that *F* is the midpoint of $\overline{CD}$. What is the ratio of the area of ΔAEF to the area of the entire parallelogram?

    a. $\frac{1}{8}$.
    b. $\frac{1}{4}$.
    c. $\frac{3}{8}$.
    d. $\frac{1}{2}$.
    e. The ratio cannot be determined uniquely from the given information.

27. A square has its base on the *x*-axis, and one vertex on each branch of the curve $y = \frac{1}{x^2}$. What is its area?

    a. $2\sqrt[3]{2}$.
    b. $2\sqrt{2}$.
    c. 2.
    d. 2.5
    e. $\sqrt{7}$.

28. Suppose that there are some blabs, some blibs, and some blubs. Suppose also that all blabs are blibs and some blubs are blabs. Which of the statements X, Y, Z **MUST** be true?

    X: All blabs are blubs.
    Y: Some blabs are not blubs.
    Z: Some blibs are blubs.

    a. X only.
    b. Y only.
    c. Z only.
    d. X and Y only.
    e. Y and Z only.

29. Let *T* be an equilateral triangle of height *h*. Let *S* be a square of side *s*. If *T* and *S* have the same area, then find $\frac{h}{s}$.

    a. $\sqrt{2}$.
    b. $\sqrt{3}$.
    c. $\sqrt[4]{3}$.
    d. $\sqrt[4]{6}$.
    e. $\frac{\sqrt{6}}{2}$.

30. A square of perimeter 20 is inscribed in a square of perimeter 28 in such a way that all vertices of the smaller square lie on the sides of the larger one. What is the greatest distance between a vertex of the inner square and a vertex of the outer square?

    a. $\sqrt{58}$.
    b. $\sqrt{65}$.
    c. 8.
    d. $\frac{7\sqrt{5}}{2}$.
    e. $5\sqrt{3}$.

31. At Central High School 50 girls play intramural basketball and 40 girls play

intramural volleyball. If 10 girls play both sports, what is the ratio of the number of girls who play only basketball to the number who play only volleyball?

a. $\frac{4}{3}$.
b. $\frac{5}{3}$.
c. $\frac{5}{4}$.
d. $\frac{5}{8}$.
e. $\frac{3}{4}$.

32. *R*, *S*, and *T* are points with $RT = 2RS$. Which of the following could be true?

I. *R*, *S*, and *T* are the vertices of a right triangle.
II. *R*, *S*, and *T* are three of the vertices of a square.
III. *R*, *S*, and *T* all lie on the circumference of a circle.

a. I only.
b. III only.
c. I and II only.
d. I and III only.
e. I, II and III.

33. To make an orange dye, 3 parts of red dye are mixed with 2 parts of yellow dye. To make a green dye, 2 parts of blue dye are mixed with 1 part of yellow dye. If equal amounts of green and orange are mixed, what is the proportion of yellow dye in the new mixture?

a. $\frac{3}{16}$.
b. $\frac{1}{4}$.
c. $\frac{11}{30}$.
d. $\frac{3}{8}$.
e. $\frac{7}{12}$.

34. Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 feet and revolves at the constant rate of one revolution per minute. How

many seconds does it take a rider to travel from the bottom of the wheel to a point 10 vertical feet above the bottom?

a. 5.
b. 6.
c. 7.5.
d. 10.
e. 15.

35. The vertices of a 3-4-5 right triangle are the centers of three mutually externally tangent circles . What is the sum of the areas of these circles?

a. 12π .
b. $\frac{25\pi}{2}$.
c. 13π .
d. $\frac{27\pi}{2}$.
e. 14π .

36. Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangles is *m* times the area of the square. The ratio of the area of the other small right triangle to the area of the square is

a. $\frac{1}{2m+1}$.

b. m.

c. $1-m$.

d. $\frac{1}{4m}$.
e. $\frac{1}{8m^2}$

37. An object in the plane moves from one lattice point to another. At each step, the object may move one unit to the right, one unit to the left, one unit up, or one unit down. If the object starts at the origin and takes a ten-step path, how many different

points could be the final point?

a. 120.
b. 121.
c. 221.
d. 230.
e. 231.

38. A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

a. 10
b. 25.
c. 36.
d. 50.
e. 60.

39. How many line segments have both their endpoints located at the vertices of a given cube?

a. 12.
b. 15.
c. 24.
d. 28.
e. 56.

40. The sides of a triangle have lengths 11, 15, and k, where k is an integer. For how many values of k is the triangle obtuse?

a. 5.
b. 7.
c. 12.
d. 13.
e. 14.