

ALGEBRA I
State Mathematics Contest Finals
April 30, 2009

1. The lines $x = \frac{1}{4}y + a$ and $y = \frac{1}{4}x + b$ intersect at the point (1,2). What is $a + b$?
- a. 0 b. $\frac{3}{4}$ c. 1 d. 2 e. $\frac{9}{4}$
2. Define the operation $*$ by $x * y = 4x - 3y + xy$, for all real numbers x and y . For how many real numbers y does $3 * y = 12$?
- a. 0 b. 1 c. 3 d. 4 e. more than 4
3. What is the probability that, of three people selected at random, at least two were born on the same day of the week?
- a. $\frac{2}{7}$ b. $\frac{3}{7}$ c. $\frac{19}{49}$ d. $\frac{27}{49}$ e. $\frac{30}{49}$
4. $ABCD$ and $DCFE$ are coplanar rectangles with $AB = 4$, $AC = 5$ and $BC = CF$. What is the length of \overline{AF} ?
- a. $2\sqrt{13}$ b. $\sqrt{34}$ c. $\sqrt{41}$ d. 6 e. 7
5. A line with slope 2 intersects a line with slope 6 at the point (40,30). What is the distance between the x -intercepts of these lines?
- a. 8 b. 10 c. 12 d. 14 e. 16
6. To complete the grid below, each of the digits 1 through 4 must occur once in each row and once in each column. What number will occupy the lower right-hand square?

- a. 1
b. 2
c. 3
d. 4
e. Cannot be determined

1		2	
2	3		
			4

7. A large rectangle is partitioned into four rectangles by two segments parallel to its sides. The areas of three of the resulting rectangles are shown. What is the area of the fourth rectangle?

- a. 10
- b. 15
- c. 20
- d. 21
- e. 25

6	14
?	35

8. *Parade* is a supplement to many Sunday newspapers. In the past year, *Parade* introduced Numbrix TM puzzles for its readers. The goal is to fill in the grid so that the numbers 1-81 are listed in sequence, such that consecutive numbers follow a horizontal or vertical path (no diagonals). Find the number to be inserted into the square marked 'X'.

9	8	7	6	61	62	67	68	69
10								70
15								73
16								74
21				X				81
22								52
27								51
28								48
29	30	33	34	43	44	45	46	47

- a. 1
- b. 39
- c. 40
- d. 57
- e. 77

9. Find $f\left(f\left(f\left(\frac{1}{2}\right)\right)\right)$ where $f(x)$ is defined as $f(x) = \begin{cases} 2x & \text{if } 0 \leq x < 1 \\ \frac{1}{2}x & \text{if } 1 \leq x < 2 \end{cases}$

- a. $\frac{1}{4}$
- b. $\frac{1}{2}$
- c. 1
- d. $1\frac{1}{2}$
- e. 2

10. Determine all values of c for which $2x^2 + 8x + c = 0$ has real solutions.

- a. $c < -8$ b. $c \leq -8$ c. $c \leq 8$ d. $c \geq 8$ e. $-8 \leq c \leq 8$

11. $|b - a| = |b| - |a|$ is true:

- a. for all real values of a and b .
b. if a and b are positive real numbers
c. for no real values of a and b .
d. If $b = 0$.
e. If $a = 0$ or $a = b$.

12. If $a > 0$ and $b > 0$ and the triangle in the first quadrant bounded by the coordinate axes and the graph $ax + by = 6$ has area 6, then $ab =$

- a. 3 b. 6 c. 12 d. 108 e. 432

13. Alice and Bob play a game involving a circle whose circumference is divided by 12 equally-spaced points. The points are numbered clockwise, from 1 to 12. Both start on point 12. Alice moves clockwise and Bob, counterclockwise. In a turn of the game, Alice moves 5 points clockwise and Bob moves 9 points counterclockwise. The game ends when they stop on the same point. How many turns will this take?

- a. 6 b. 8 c. 12 d. 14 e. 24

14. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$, then if $\begin{vmatrix} 2 & x \\ 3 & 5 \end{vmatrix} = \begin{vmatrix} x & 2 \\ 1 & 1 \end{vmatrix}$, $x =$

- a. -3 b. 1 c. 0 d. 3 e. 4

15. If $a + b = 1$, and $a^2 + b^2 = 2$, then the product of a and b equals

- a. $-\frac{1}{2}$ b. 1 c. $\frac{1}{2}$ d. $1\frac{1}{2}$ e. 2

16. If the binary operation $*$ is defined for integers by $a * b = a + b - ab$, which of the following is (are) true?

- I. $*$ is commutative.
II. $*$ is associative
III. There exists some integer which is an identity for $*$

- a. I only b. III only c. I and II only d. I and III only e. I, II and III

17. $|x - 4| > |x + 4|$ when
- a. $x > 0$ b. $x < 4$ c. $x < 0$ d. $x > 4$ e. none of these.
18. Thirty dollars is to be split among 8 people. Each person gets at least \$1; at least one person gets more than \$5; at least four other people get more than \$1. If all 8 people get an exact number of dollars, the largest amount (in dollars) that could be received by a person is
- a. 13 b. 15 c. 17 d. 19 e. 23
19. If $(101)(99)$ is expressed as $(0.1n + 1)(0.1n - 1)$, where $n > 0$, what is the value of n ?
- a. 9 b. 10 c. 90 d. 100 e. 1000
20. The average age of 5 people in a room is 30 years. An 18-year-old person leaves the room. What is the average age of the four remaining people?
- a. 25 b. 26 c. 29 d. 33 e. 36
21. Bill and Jill are hired to paint a line on a road. If Bill works by himself, he could paint the line in B hours. If Jill works by herself, she could paint the line in J hours. Bill starts painting the line from one end, and one hour later Jill begins painting the line from the other end. They both work until the line is painted. Which of the following is an expression for the number of hours that Bill works?
- a. $\frac{B(J + 1)}{B + J}$ b. $J + 1$ c. $\frac{BJ}{B + J} + 1$ d. $\frac{B + J - 1}{2}$ e. $\frac{B(J - 1)}{B + J}$
22. How many two-digit positive integers N have the property that the sum of N and the number obtained by reversing the order of the digits of N is a perfect square?
- a. 4 b. 5 c. 6 d. 7 e. 8
23. Suppose that $f(x) = ax + b$, where a and b are real numbers. Given that $f(f(f(x))) = 8x + 21$, what is the value of $a + b$?
- a. 2 b. 3 c. 4 d. 5 e. 6
24. Of the numbers 6^{100} , 5^{200} , 4^{300} , 3^{400} , 2^{500} which is the greatest?
- a. 6^{100} b. 5^{200} c. 4^{300} d. 3^{400} e. 2^{500}

25. In a magic square the sum of the numbers in any row, column, or diagonal is the same. For the magic square below, the sum of the numbers in any row is

$2x$	3	2
		-3
0	x	

- a. 0 b. 1 c. 3 d. 7 e. 9
26. If x and y are positive numbers and the average of 4, 20 and x is equal to the average of y and 16, then the ratio $x : y$ is:
- a. 3:2 b. 2:3 c. 1:1 d. 2:5 e. 5:2
27. The whole numbers m and n satisfy $m + n = 20$ and $\frac{1}{m} + \frac{1}{n} = \frac{5}{24}$. The product mn is
- a. 72 b. 36 c. 48 d. 96 e. 24
28. If $y = a(x - 2)^2 + c$ and $y = (2x - 5)(x - b)$ represent the same quadratic function, the value of b is:
- a. 3 b. $\frac{3}{2}$ c. $\frac{4}{5}$ d. $-\frac{5}{2}$ e. $\frac{8}{5}$
29. A computer software retailer has 1200 copies of a new software package to sell. From past experience she knows that
- I. Half of them will sell right away at the original price she sets.
 - II. Two-thirds of the remainder will sell later when their price is reduced by 40%, and
 - III. The remaining copies will sell in a clearance sale at 75% off the original price.

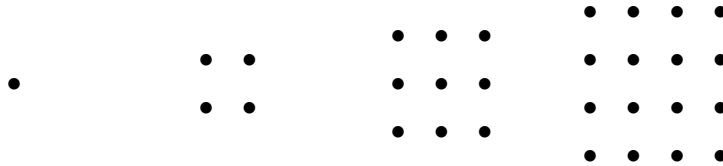
In order to make a reasonable profit, the total sales revenue must be \$72,000. To the nearest cent, what original price should she set?

- a. \$60.01 b. \$75.01 c. \$79.13 d. \$80.90 e. \$240.01

30. Jonathan sold two mountain bikes. He made a 30% profit on the first one and a 50% profit on the second one. If his total profit was 45%, then what was the ratio of his cost for the first bike to his cost for the second bike?

- a. 1:2 b. 1:3 c. 1:5 d. 2:3 e. 2:5

31. Consider the following pattern



What is the difference between the number of dots in the n^{th} figure and the number of dots in the $(n-1)^{\text{th}}$ figure?

- a. $2n$ b. $n-1$ c. $2n-1$ d. n^2-1 e. $(n-1)^2$

32. Of the following numbers, the one that can be written in the form $3N$, where N is an integer, is:

- a. 44 b. 4,444 c. 44,444 d. 444,444 e. 4,444,444

33. A box contains 4 fair coins and 6 biased coins. Whenever a fair coin is flipped, it comes up heads with probability 0.5. Whenever a biased coin is flipped, it comes up heads with probability 0.8. A coin is randomly chosen from the box and then flipped. What is the probability that it will come up heads?

- a. 0.6 b. 0.64 c. 0.68 d. 0.72 e. 0.76

34. What is the value of k , so that the graph of $4x + ky = 8$ is a line parallel to the line passing through the points $(6,-2)$ and $(4,-1)$?

- a. -8 b. $\frac{1}{2}$ c. $-\frac{1}{2}$ d. 2 e. 8

35. If $|x-2| = p$, where $x < 2$, then $x-p =$

- a. -2 b. 2 c. $2-2p$ d. $2p-2$ e. $|2p-2|$

36. Gail has an incredible coin changing machine. When she puts in a quarter, it returns five nickels; when she puts in a nickel, it returns five pennies; and when she puts in a penny, it returns five quarters. Gail starts with just one penny. Which of the following amounts could Gail have after using the machine repeatedly?
- a. \$3.63 b. \$5.13 c. \$6.30 d. \$7.45 e. \$9.07
37. An unusual die has the numbers 2, 2, 3, 3, 5 and 8 on its six faces. Two of these dice are rolled and the two numbers on the top faces are added. How many different totals are possible?
- a. 8 b. 9 c. 10 d. 11 e. 12
38. The terms a_1, a_2, a_3 form an arithmetic sequence whose sum is 18. The terms $a_1 + 1, a_2, a_3 + 2$, in that order, form a geometric sequence. Find the sum of all possible values of a_1 .
- a. 0 b. 2 c. 7 d. 11 e. 13
39. When 60 minutes elapse on a correct clock, 62 minutes register on clock F (fast) and only 56 minutes register on clock S (slow). If later in the day clock F reads 8:00 and clock S reads 7:00, what was the correct time when the two clocks were originally set?
- a. 4:20 b. 6:40 c. 7:20 d. 8:20 e. 9:40
40. Ramona just completed writing her first book and numbered the pages. Numbering the book required 2649 digits. How many pages does Ramona's book have?
- a. 913 b. 917 c. 919 d. 923 e. 929