Algebra II
State Mathematics Contest Finals
May 2, 2002

1. If $4^x + 4^x + 4^x + 4^x + 4^x + 4^x + 4^x + 4^x = \frac{1}{512}$, what is the value of $-\frac{3}{x}$?

a) 0.50 b) 0.75 c) -0.75 d) -4.25 e) 4.25

2. A point (x, y) with integral coordinates is called a lattice point. How many lattice points lie on the graph of $x^2 + y^2 = 25$?

a) 6 b) 8 c) 12 d) 16 e) 13

3. Given $q^2 - 3q + 5 = 0$, determine the value of $q^4 - 6q^3 + 9q^2 - 7$.

a) 32 b) 14 c) 16 d) 25 e) 18

4. Suppose $g(x) = \cos x$ and *a* is the smallest positive number so that the graph of $y = g(x - a)$ will be symmetric about the y-axis. Determine the value of $\cos a$.

a) $-\pi$ b) -1 c) π d) 1 e) $\frac{\pi}{2}$

5. Find $\log_{\sqrt{7}} 343$.

a) 4 b) 5 c) 6 d) 7 e) impossible to determine

6. Simplify: $\sqrt{2\sqrt{63} + \frac{2}{8 - 3\sqrt{7}}}$

a) $2\sqrt{4+\sqrt{7}}$ b) $\sqrt{65}$ c) $\sqrt{\sqrt{28}}$ d) $2\sqrt{4+3\sqrt{7}}$ e) $\sqrt{2-4\sqrt{7}}$

7. Let $F(x)$ denote a polynomial of degree 3. Suppose $F(x+2) - F(x) = x^2 + 4x + 4$ and $F(1) = 9$. Find the value of *F*(3).

a) 18 b) 19 c) 23 d) 13 e) 17

8. Evaluate $\left(\frac{i^{123}}{i^{6}}\right)^{-5}$ given that $i=\sqrt{-1}$.

a) $-i$ b) i c) -1 d) 1 e) 0

9. Consider the sequence $1,-2,\ 3,-4,\ 5,-6,\ldots,\ n\cdot(-1)^{n+1}$. What is the average of the first 300 terms of the sequence?

a) -1 b) 0.5 c) 0 d) -0.5 e) 1

10. If *x* is 60% larger than *z* and *y* is 25% larger than *z*, then *x* is what percent larger than *y*?

a) 28% b) 25% c) 55% d) 100% e) 78%

11. Suppose you are sitting at a distance *d* from a stereo speaker. You and the speaker are outside. The loudness *L* of the sound is inversely proportional to the square of *d*. What happens to the sound if you move three times the distance *d* from the speaker?

a) The loudness increases by a factor of 1/9.
b) The loudness increases by a factor of 3.
c) The loudness decreases by a factor of 1/9.
d) The loudness decreases by a factor of 3.
e) There is no change in the loudness.

12. Find the sum of all values of *x* for which ${}_{10}C_{x} = {}_{10}C_{3x-6}$ where ${}_{n}C_{r}$ means the combination of n things taken r at a time.

a) 12 b) 7 c) 3 d) 10 e) none of these

13. Find m if $\sqrt{\frac{a}{b}\sqrt{\frac{b}{a}\sqrt{\frac{a}{b}}}}=\left(\frac{a}{b}\right)^{m}$.

a) $\frac{5}{8}$ b) $\frac{8}{5}$ c) $\frac{1}{8}$ d) $\frac{8}{3}$ e) $\frac{3}{8}$

14. The intersection of planes defined by the equations $x+2y=0$ and $6x+y+2z=-1$ contains the point $(w, w^{2}, w^{3}+1)$. Find the value $w^{3}+w^{2}+w+1$.

a) $-\frac{1}{3}$ b) $\frac{3}{5}$ c) $-\frac{1}{12}$ d) $\frac{5}{8}$ e) $\frac{3}{7}$

15. If $p = a + 3b + 2c$ with $a, b, c \in$ Whole #'s, $a \neq 0$, find the smallest value of p that will make abc_{13} divisible by 2 but not by 4. (abc_{13} represents a three digit number in base 13.)

a) 2 b) 3 c) 4 d) 6 e) none of these

16. Determine the value of m such that the three lines $y = 3x + 2$, $y = -2$, and $y = mx - 3$ are concurrent.

a) $-\frac{3}{2}$ b) $\frac{3}{4}$ c) $\frac{2}{3}$ d) $\frac{4}{3}$ e) $-\frac{3}{4}$

17. When an integer N is divided by 2795, the remainder is 66. What is the remainder when the integer N is divided by 43?

a) 21 b) 18 c) 23 d) 15 e) 17

18. The decimal form of $S = \frac{1}{10} + \frac{2}{100} + \frac{3}{1000} + \frac{4}{10000} + \cdots$ will never contain which of the following digits?

a) 9 b) 6 c) 8 d) 7 e) all digits are possible

19. If $\left(\frac{4}{9}\right)^{\sqrt{x}} = \left(\frac{2}{3}\right)^{39-\sqrt{x}}$, find the product of the digits in the numeral x.

a) 54 b) 15 c) 26 d) 42 e) 18

20. Determine the coefficient of x^7 in the expansion of $\left(\frac{x^2}{2} - \frac{2}{x}\right)^8$.

a) 12 b) -12 c) 13 d) -14 e) 14

21. Find the equation of the perpendicular bisector of the line segment whose endpoints are $(6, 10)$ and $(8, -4)$. Express the equation in the form of $px + qy = 1$. What is the value of $p - 3q$?

a) $-\frac{3}{2}$ b) $-\frac{7}{11}$ c) $\frac{3}{2}$ d) $-\frac{11}{7}$ e) $\frac{11}{7}$

22. Given: $\log_9 \sqrt[8]{729 \cdot \sqrt[4]{x \cdot \sqrt{243 \cdot 81^{-0.75}}}} = \frac{1}{2}$. Find the sum of the digits of the value x.

a) 12 b) 15 c) 16 d) 17 e) 18

23. Simplify the expression $5^{(3\ln e)(\log_5 a)}$.

a) 5^{3a} b) 5^{a^3} c) e d) a^3 e) $3a$

24. Find the sum of the roots of $|x-7|^2 + 2|x-7| = 24$.

a) 4 b) 14 c) 7 d) 12 e) 15

25. If -3 and $\frac{1}{2}$ are two of the three zeros of the function $f(x) = ax^3 + 3x^2 - bx + 3$ with $a, b \in$ Real #'s, find the numerical value of $a + b$.

a) 6 b) 7 c) 8 d) 9 e) 10

26. Given: $g(x) = ax^4 - 14x^3 + a^2x^2 - 9x + \frac{a}{2}$. For what integer value of a will $g(x)$ be exactly divisible by $x - 2$?

a) -4 b) 2 c) 4 d) -2 e) 6

27. A small tree 5 feet from a lamp post casts a shadow 4 feet long. If the lamp post were 2 feet higher the shadow would only be 3 feet long. How tall is the tree?

a) $\frac{7}{2}$ ft. b) $\frac{24}{5}$ ft. c) $\frac{8}{3}$ ft. d) $\frac{14}{3}$ ft. e) $\frac{17}{5}$ ft.

28. A circle is inscribed in a triangle with sides of lengths of 8, 17, and 19. Let the segments determined by the point of tangency on the side of length 8 be *w* and *r*, with $w < r$. Find the ratio of *w* to *r*.

a) 3:5 b) 5:7 c) 2:3 d) 1:5 e) 7:9

29. If $f(x)=\frac{a}{x-4}$, $g(x)=\frac{b}{x}$, $(f\circ g)(-1)=-\frac{1}{2}$, and $(f\circ g)(1)=-\frac{3}{2}$, find the value of $(a+3b)$.

a) 6 b) 7 c) 8 d) 9 e) 10

30. The value of q which will make the roots of $2x^2-27x+q=0$ be in the ratio of 1:2 is in the interval:

a) [0, 10) b) [10, 25) c) [25, 50) d) [50, 75) e) [75, 90)

31. Every hour on the hour a train leaves A-boro for B-boro, while the trains going in the other direction leave the B-boro station every hour on the half-hour. The trip between the two cities takes exactly two hours. After a train leaves from A-boro to B-boro, how many of the trains going in the opposite direction will it meet during the trip?

a) 2 b) 3 c) 4 d) 5 e) 6

32. If $t<-2$, then $\left|1-\left|t+1\right|\right|$ equals

a) $-2+t$ b) $-t$ c) t d) $2+t$ e) $-2-t$

33. The vertex of the graph of the parabola $y=3x^2+bx+c$ is at $(-2,-7)$; determine the value of $b-c$.

a) 7 b) -10 c) -4 d) 6 e) 9

34. For all real x, $f(x)=x+3$ and $g(x)$ is a polynomial of degree 2 such that $g(f(x))=x^2+2$. Find $g(-2)$.

a) 8 b) 16 c) 27 d) 18 e) 13

35. If roses cost *p* cents a dozen less, one would pay 2 cents less for *p* + 1 roses than if they cost *p* cents a dozen more. Find the value of *p*.

a) 6 b) 3 c) 7 d) 9 e) 2

36. The difference between the square of the arithmetic mean of two numbers and the square of their geometric mean is 49. Find the difference between the arithmetic mean of their squares and the square of their arithmetic mean.

a) 98 b) 40 c) 45 d) 49 e) 55

37. If $a\log_{200} 5 + b\log_{200} 2 = c$ with $\gcd(a,b,c) = 1$, find the value of $a + 2b - c$.

a) 7 b) 6 c) 9 d) 11 e) 8

38. Find the value of x where (x, y) is the solution to the system of equations:

$$2(\cos f)\,x - 5(\sin f)\,y = \sec f$$
$$2(\sin f)\,x + 5(\cos f)\,y = 3\csc f$$

a) 2 b) 1 c) $4\cos f$ d) $\sin f$ e) none of these

39. If $\log_{\sin x}(\cos x) = \frac{1}{2}$ and $0 < x < \frac{p}{2}$, find the value of $\sin x$.

a) $\frac{1}{3}\sqrt{3}$ b) $\frac{1}{2}(\sqrt{5}-1)$ c) $\frac{1}{2}(\sqrt{5}+1)$ d) $\frac{2}{3}\sqrt{3}$ e) $\sqrt{3} - \frac{1}{2}$

40. Suppose the lengths of three of the four lateral edges of a pyramid with a rectangular base are 8, 5, and 1 in that order. Find the length of the fourth lateral edge.

a) $\sqrt{79}$ b) $\sqrt{47}$ c) $2\sqrt{10}$ d) $2\sqrt{11}$ e) $2\sqrt{3}$