State Mathematics Contest: Geometry

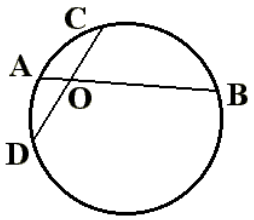
May 2, 2002

1. Find the area of a regular hexagon whose vertices lie on a circle with radius 6 cm.

   a. 36π cm^2 b. $54\sqrt{3}$ cm^2 c. $3\pi\sqrt{3}$ cm^2 d. $36\sqrt{3}$ cm^2 e. none of these

2. Two chords $\overline{AB}$ and $\overline{DC}$ intersect each other so that $AO = 2$, $DC = 8$, and $OC = DO$. How long is $\overline{AB}$?

   a. 8 units
   b. $2\sqrt{5}$ units
   c. $\sqrt{38}$ units
   d. 10 units
   e. none of these



3. Given the following two circles, find the algebraic equation of the line connecting their centers.

   Circle I: $x^2 - 4x + y^2 + 6y = 12$ Circle II: $x^2 + y^2 = 8x$

   a. $3x + 2y = 12$ b. $-4x + 6y = 12$ c. $x + 2y = -4$
   d. $2x + y = -8$ e. none of these

4. Suppose we have an arbitrary triangle with line segments that bisect each angle and extend to the opposite side, which of the following statements are necessarily true.

   I. Each angle bisector also bisects the opposite side of the triangle.
   II. Each angle bisector divides the triangle into two equal areas.
   III. The three angle bisectors all intersect at the same point.

   a. I only b. II only c. III only d. I and III e. II and III

5. Let A, B, and C be nonempty sets and $\overline{A}$ be defined as the complement of set A (similar notation for other sets). If $A \subset \overline{B}$, $x \in B$, and $x \in \left(\left(A \cap \overline{C}\right) \cup \left(\overline{A} \cap C\right)\right)$ then:

a. $x \in \left(\overline{A} \cap B \cap C\right)$ b. $x \in \left(A \cap B \cap \overline{C}\right)$ c. $x \in \left(A \cap B \cap C\right)$
d. $x \in \left(A \cap \left(B \cup C\right)\right)$ e. $x \in \left(\overline{A} \cap \left(B \cap \overline{C}\right)\right)$

6. If the height of a cylindrical can is increased by 28%, by approximately what percentage should the diameter be increased in order to double the volume of the can?

a. 72% b. 31.1% c. 28.8% d. 25% e. 23.1%

7. The following statements describe a race between Art, Bobbie, Chris, and Francis.
   i. Art is not last but is 8 seconds behind the leader.
   ii. Bobbie is 5 seconds ahead of the next runner.
   iii. Chris is 1 second behind someone.
   iv. Francis is ahead of Bobbie.

   How many seconds are there between the leader and the second place runner?

a. 8 seconds b. 5 seconds c. 3 seconds d. 2 seconds e. 1 second

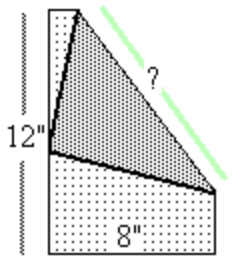
8. Find the circumference of a circle that circumscribes a regular hexagon with area of $42\sqrt{3}\ \text{cm}^2$.

a. $4\pi\sqrt{7}$ cm b. $21\pi\sqrt{3}$ cm c. $2\pi\sqrt{42}$ cm d) $4\pi\sqrt{42}$ cm e. none of these

9. A ship travels west for 16 miles, then northwest for 12 miles, finally it goes north for 9 miles, approximately how far is it from its starting point?

a. 37 miles b. 30 miles c. 27 miles d. 22 miles e. none of these

10 . An 8 inch by 12 inch piece of paper is folded so that the upper right corner touches the middle of the opposite (left) side. Find the length of the fold.



a. $4\sqrt{13}$ inches b. 10 inches c. $\sqrt{130}$ inches

d. $10\frac{5}{12}$ inches e. $\sqrt{12\sqrt{2}+16\sqrt{3}}$ inches

11. Find an expression for the area of a rectangle in terms of the length of its diagonal, *d*, and its perimeter, *P*.

a. $\frac{Pd}{4\sqrt{2}}$ b. $\frac{P^2-4d^2}{8}$ c. $\frac{(P-2d)^2}{8}$ d. $\frac{P\left(\sqrt{P^2+4d}\right)}{16}$ e. none of these

12. Karen knows her old analog clock (one that only gives the hours from 1 - 12) is fast. By coincidence when she left for a 20-day vacation the clock's time was accurate, and when she came back after exactly 20 days the clock's time was also accurate. Given that the clock is fast, what is the least amount of minutes its minute-hand could move in one hour?

a. 63 minutes b. 62 minutes c. $61\frac{1}{2}$ minutes d. 61 minutes e. $60\frac{1}{2}$ minutes

13. A square of unknown size has 12 cm added to its perimeter to make a new square. If this new square has twice the area of the old square, what is the length of the new square's sides?

a. $3+3\sqrt{2}$ cm b. $6\sqrt{2}$ cm c. $6+3\sqrt{2}$ cm d. 6 cm e. none of these

14. Given two sequences:

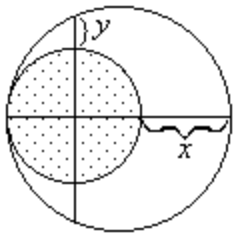
Sequence A: $a_1=1$, $a_2=3$, $a_3=6$, $a_4=10$, $a_5=15$, ... a_m

Sequence B: $b_1=1$, $b_2=4$, $b_3=9$, $b_4=16$, ... b_n

Find the smallest positive value for m - n so that $a_m = b_n$.

a. 6 b. 4 c. 3 d. 2 e. 1

15. Two circles are drawn internally tangent to one another, as shown. The segments given are parts of the extended diameters of the smaller circle. If $x=12$ and $y=5$ what is the length of the larger circle's radius.



a. $18\frac{1}{2}$
b. $12\frac{1}{2}$
c. 17
d. $2\sqrt{60}$
e. $21\frac{1}{4}$

16. Find all possible values for y given that x, y, z are natural numbers and the following constraints:

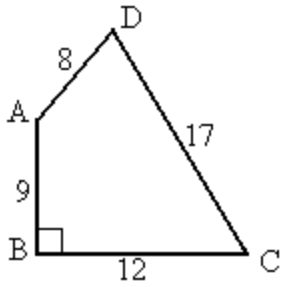
i. $y > x+3$
ii. $0 < z-y < 10$
iii. $x+z=13$

a. $y \in \{4,5,6\}$ b. $y \in \{8,9,\cdots,11\}$ c. $y \in \{4,5,\cdots,9\}$ d. $y \in \{5,6,..,11\}$ e. $y \in \{5,6\cdots,12\}$

17. A circle of radius 25 has a chord going through a point that is located 10 units from the center. What is the shortest possible length that chord could have?

a. 25 b. $\sqrt{525}$ c. 40 d. $\sqrt{1050}$ e. $\sqrt{2100}$

18. Find the area of quadrilateral ABCD with lengths as shown and the measure of $\angle ABC = 90°$.



a. 46 sq units b. 130 sq units c. 114 sq units
d. 168 sq units e. none of these

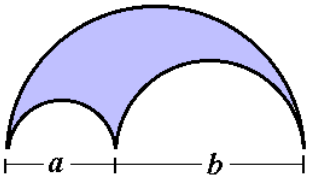
19. Find the size of the larger angle in a parallelogram given that the smaller angle is one quarter the size of the larger angle.

a. 72° b. 80° c. $122\frac{1}{4}°$ d. 144° e. none of these

20. Find the area of a right triangle whose perimeter is 28 cm and whose hypotenuse is 12 cm.

a. $48\ \text{cm}^2$ b. $24\sqrt{7}\ \text{cm}^2$ c. $12\sqrt{7}\ \text{cm}^2$ d. $8+4\sqrt{2}\ \text{cm}^2$ e. $28\ \text{cm}^2$

21. An *arbelos*, pictured on the right, is the region formed by three mutually tangent circles whose centers are collinear. If the diameters of the two smaller circles are a and b, what is the circumference of the *arbelos*?

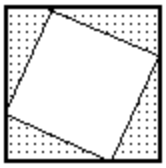


a. $(a+b)p$ b. $2p\sqrt{ab}$ c. $\dfrac{abp}{2}$ d. $2p\sqrt{a^2+b^2}$ e. $\dfrac{3(a+b)p}{2}$

22. Find the area of a rectangle whose perimeter is the square of its longer side and 10 times the length of its shorter side.

a. 1,000 b. 25 c. $\dfrac{25}{16}$ d. $\dfrac{25}{8}$ e. $\dfrac{5\sqrt{10}}{8}$

23. The area of the shaded region between two squares is half as large as the area of the smaller square. If the smaller square has an area of $42\ \text{cm}^2$, how long is the side the larger square?



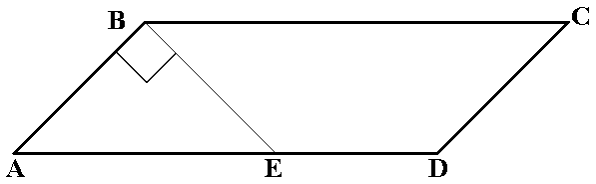
a. $3\sqrt{7}$ cm b. 8 cm c. $7\sqrt{3}$ cm d. $\sqrt{42}$ cm e. none of these

24. A wheel with a 30 centimeter radius rolled down a hill at a constant speed of 20 kilometers per hour. If it rolled for 10 minutes how many complete revolutions did it make?

a. 1,768 b. 3,333 c. 3,536 d. 106 e. 106,103

25. The parallelogram ABCD has $AB = BE = ED$, and $m\angle ABE = 90°$. If its area is 25 how long is side $\overline{AD}$?

a. $5\left(\sqrt{2}+1\right)$
b. $5\left(\sqrt[4]{10}\right)$
c. $5\left(\sqrt[4]{2}+\sqrt{2}\right)$
d. $10\left(\sqrt{2}+1\right)$
e. $5\sqrt{2+\sqrt{2}}$

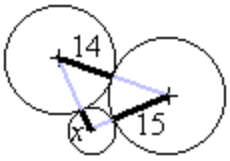


26. Given two intersecting lines $2x - y = 1$ and $3x - 2y = -1$, which of the lines listed below also passes through the point of intersection?

a. $y = 2x + 1$ b. $y = \frac{3}{2}x - 1$ c. $5x - 3y = 1$ d. $3x + 5y = 15$ e. none of these

27. Three circles, mutually tangent to one another have radii of lengths, 14, 15, and x. If their centers are vertices of a right triangle, find the value for x.

a. 5 b. $\sqrt{29}$ c. 6 d. $\sqrt{421}$ e. none of these



28. In a large bin there are 118 hats. They come in two colors, red or black, and two styles with or without stripes. If there are 44 red hats, 64 hats with stripes, and 33 red hats with stripes, how many black hats without stripes are there?

a. 85 b. 43 c. 23 d. 10 c. 41

29. The height of a square pyramid formed by four equilateral triangles is 10. What is the surface area of one of these triangles?

a. $50\sqrt{3}$ b. 100 c. $3\sqrt{50}$ d. $100\sqrt{3}$ e. none of these

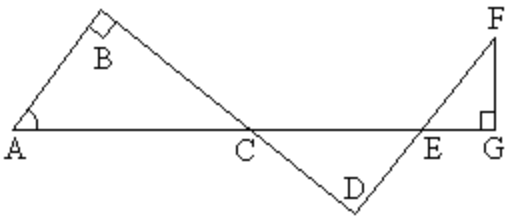
30. A cylindrical can with a diameter of 14 cm and a height of 28 cm contains 2 spherical balls each with a diameter of 14 cm. What percentage of the space in the can is not occupied by the balls?

a. $33\frac{1}{3}\%$ b. 25% c. $16\frac{2}{3}\%$ d. $66\frac{2}{3}\%$ e. $8\frac{1}{3}\%$

31. A right triangle with sides 9, 40, and 41 has its shortest side quadrupled in length, without changing the area and still maintaining the right angle. What is the length of the new triangle's hypotenuse?

a. $10\frac{1}{2}$ b. $2\sqrt{349}$ c. $20\frac{1}{2}$ d. $4\sqrt{106}$ e. none of these

32. Given the figure on the right where $\angle ABC$ and $\angle EGF$ are right angles, $m\angle CAB = 54°$, and $m\angle CDE = 104°$, find the measure of $\angle EFG$.



a. $54°$ b. $36°$ c. $50°$ d. $40°$ e. none of these

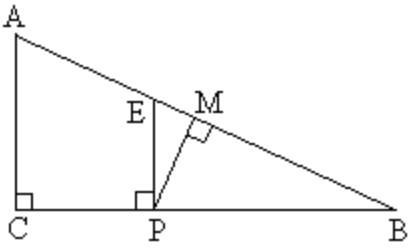
33. Find the area of a circle that is inscribed in a right triangle with sides of length: 8, 15 and 17.

a. 9π b. $11\frac{1}{9}\pi$ c. 14π c. 10π e. none of these

34. A line parallel to the base of a triangle cuts the triangle into two regions of equal area. This line also cuts the altitude into two parts. Find the ratio of the two parts of the altitude.

a. 1 : 1 b. 1 : 2 c. $1:\sqrt{2}$ d. $1:\sqrt{2}+1$ e. none of these

35. Given the right triangle ABC as shown with $\overline{EP} \perp \overline{CB}$ $\overline{PM} \perp \overline{AB}$, and M as the midpoint of $\overline{AB}$.
If $AC = 6$ and $CB = 8$, what is the length of $\overline{EP}$?



a. 5 b. $\sqrt{14}$ c. $\frac{75}{16}$ d. 4 e. none of these

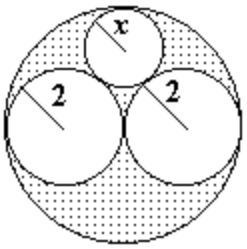
36. Approximate the circumference of a circle whose area will double when the radius is increased by 2.

a. 42.9 b. 146.5 c. 30.3 d. 93.8 e. 12.6

37. A rectangular solid with a square base and a height that is half its width has a surface area of 1156 square feet. Find its volume.

a. 2,318 cu ft. b. 4,913 cu ft. c. $544\sqrt{17}$ cu ft. d. 39,304 cu ft. e. 2,456.5 cu ft.

38. Two circles each with radius 2 are inscribed into a larger circle of radius 4 so that each is mutually tangent to the other. (See figure) Find the radius of a circle that is tangent to the three other circles.



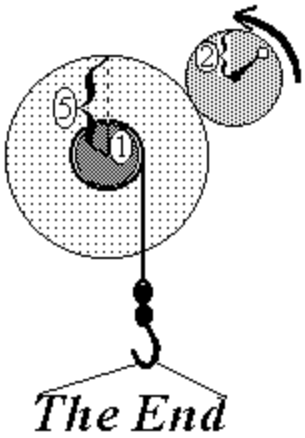
a. 1 b. $\frac{\sqrt{5}}{2}$ c. $\frac{4}{3}$

d. $2\left(1-\sqrt{2}\right)$ e. none of these

39. A string is wrapped around a cylinder in such a way that it starts at the bottom wraps around the cylinder three times and ends up on top. If the cylinder has a diameter 10 cm and a height of 30 cm what is the shortest possible length of the piece of string?

a. $\sqrt{910p}$ cm b. $30p\sqrt{5}$ cm c. $30p + 30$ cm d. $30\sqrt{\pi^2+1}$ cm e. $90p$ cm

40. Two wheels drive a wench as shown in the figure on the right. The wheel with the crank has radius 2 inches. It drives a wheel of radius 5 inches and at the larger wheel's center is a spool with a 1 inch radius. Approximately how many turns of the crank are needed to lower the hook 14 inches?



a. 2.23 b. 0.89 c. 3.14 d. 11.67 e. 5.57

The End