

Algebra I
State Mathematics Contest Finals, May 1, 2003

1. Solve for x , given: $y = x - 1$ and $y = 2x + 1$.

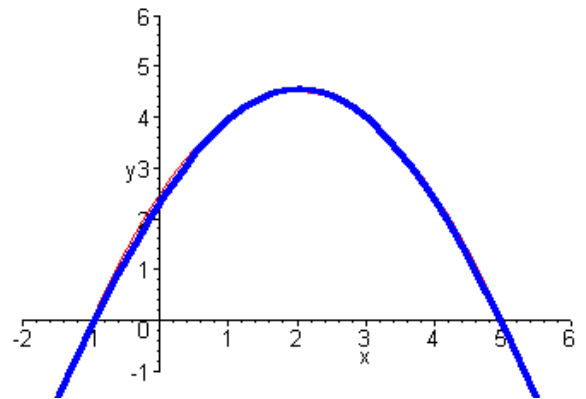
- a) $x = 1$ b) $x = -1$ c) $x = -2$ d) $x = -3$ e) None of these

2. Which of the following expressions is equivalent to $(a^{-1} - 2b)^2$?

- a) $\frac{1 - 4ab + 4a^2b^2}{a}$ b) $\frac{1 - 4b + 4b^2}{a^2}$ c) $\frac{1}{a^2} - 4b^2$
 d) $\frac{1}{a^2} - \frac{4b}{a} + 4b^2$ e) $-a^2 + 4ab + 4b^2$

3. Which function best describes the graph given on the right.

- a) $f(x) = -x^2 + x + 5$
 b) $f(x) = -\frac{1}{2}(x^2 - 4x - 5)$
 c) $f(x) = \frac{1}{2}(x^2 - 4x + 5)$
 d) $f(x) = 9 - (x - 2)^2$
 e) $f(x) = -\frac{1}{2}(x^2 + 4x - 5)$



4. Given $f(x) = \frac{x-1}{x+1}$ which expression is equivalent to $f(1-x)$?

- a) $\frac{x}{x-2}$ b) $\frac{x-2}{1-x}$ c) $\frac{4-4x-x^2}{x+1}$ d) $\frac{x-1}{x^2+1}$ e) None of these

5. Find all solutions to the following equation: $x^3 - 6x = x^2$.

- a) $x \in \{-2, 3\}$ b) $x \in \{0, 2, 3\}$ c) $x \in \{0, 2, -3\}$ d) $x \in \{0\}$ e) $x \in \{0, -2, 3\}$

6. Find the volume of a cube whose surface area has as many square centimeters as the total lengths of its edges has centimeters.

- a) 2 cm^3 b) 8 cm^3 c) $3\sqrt{3} \text{ cm}^3$ d) 27 cm^3 e) 216 cm^3

7. Find the next two number in this sequence: 245, 125, 65, 35, ...

- a) ..., 20, 12.5, ... b) ..., 20, 10, ... c) ..., 25, 15, ... d) ..., 25, 20, ... e) ..., 15, 5, ...

8. Find the equation of a line parallel to $3x - 5y = 15$ that intersects the line $y = x$ at $x = 3$.

- a) $y = \frac{5}{3}x - 2$ b) $y = -\frac{5}{3}x + 8$ c) $y = \frac{3}{5}x + \frac{6}{5}$ d) $y = \frac{3}{5}x - 3$ e) $y = 3x - 6$

9. Solve for x . $\frac{x+c}{x+1} = c+1$

- a) $x = c$ b) $x = c - 2$ c) $x = \frac{c-1}{c}$ d) $x = \frac{c-1}{c+1}$ e) $x = \frac{-1}{c}$

10. Chris's monthly allowance starts with \$10 in January, \$20 in February, and continues increasing by \$10 each month. Pat's allowance starts at 10¢ and doubles each month. In which month did Pat's allowance begin to exceed Chris's?

- a) 11th month b) 12th month c) 13th month d) 15th month e) 20th month

11. Given $y \leq 8 - 0.2x$ and $y \leq 15 - 3x$ Determine the maximum value of $x + y$.
- a) 2.5 b) 8 c) 10 d) 15 e) There is no maximum.
12. Let $A, B, C, D > 0$ where $A = B + C$, $C = D + B - A$, and $D = 2B - A$. Arrange the four numbers in increasing order.
- a) $ABCD$ b) $CDBA$ c) $DCBA$ d) $CBDA$ e) $BDCA$
13. Find the distance between the two intersections of the line, $y = 2x - 3$, and the parabola, $y = x^2 + 2x - 7$.
- a) $\sqrt{13}$ b) 4 c) $\sqrt{53}$ d) 8 e) $\sqrt{80}$
14. Given $\frac{4}{2^x} = \sqrt{8^x}$. Solve for x .
- a) $\frac{2}{5}$ b) $\frac{8}{5}$ c) $\frac{4}{3}$ d) $\frac{-3}{5}$ e) None of these
15. Jerry, who walks 2.4 miles per hour, starts walking down a trail at 7am in the morning. Travis, starts an hour later on the same trail, and catches up with Travis at noon. How fast did Travis walk?
- a) 3 mph b) $3.5\bar{3}$ mph c) 4.2 mph d) 4.8 mph e) None of these
16. Given that there are 4 guys and 2 girls that must be split into 3 pairs, and assume that the splitting into pairs is done randomly. What is the probability that the two girls are in one pair?
- a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{5}$ d) $\frac{1}{5}$ e) $\frac{1}{6}$

17. Luther has the following grades: 83, 85, 93, 91. What minimum grade does he need on the next exam to raise his average above 89.5? (Only whole numbers are possible scores.)
- a) 88 b) 90 c) 91 d) 95 e) 96
18. For what value of c are the following three points collinear: $(1, 5)$, $(3, c)$ and $(7, 14)$.
- a) $c = 8$ b) $c = 9.5$ c) $c = 7$ d) $c = 6.3\bar{3}$ e) None of these
19. $x + y + z = 20$, $x - y = 6$ and $y - z = 4$ Find x .
- a) 12 b) 2 c) 6 d) 10 e) None of these
20. Let $y = 10 + 3x - x^2$ where x and y are positive whole numbers. Find the sum of all possible values of y .
- a) 10 b) 15 c) 40 d) 66 e) 72
21. It took Peter 5 hours of driving to reach his destination. If he drove 55 mph half of the distance and 70 mph the second part, how far did he travel?
- a) $305\frac{13}{77}$ miles b) 308 miles c) 312.5 miles d) $327\frac{5}{7}$ e) None of these
22. Find the sum of two numbers with the following properties: The square of the first number is 10 units larger than the second, and doubling the first number is 7 units larger than the second.
- a) -1 or 3 b) -9 or -1 c) -10 or 2 d) -10 or -16 e) None of these

23. Two points on the line $y = 2.4x - 1.6$ are exactly 13 units apart. If both points are in the first quadrant and one of the points is (4, 8), find the other point.

- a) (16, 13) b) (14, 32) c) $(7 + 0.5\sqrt{7}, 15.2 + 1.2\sqrt{7})$ d) (9, 20) e) (10, 22.4)

24. Define \oplus as: $a \oplus b = \frac{1}{a} + \frac{1}{b}$ where a and b are non-zero rational numbers. Evaluate: $(x \oplus 2) \oplus (x + 2)$

- a) $2x + 4$ b) $\frac{x^2 + 6x + 4}{2x(x + 2)}$ c) $\frac{2}{x + 2}$ d) $\frac{x^2 + 4x + 6}{x + 2}$ e) $\frac{2x + 1}{x + 2}$

25. Define \oplus as above: $a \oplus b = \frac{1}{a} + \frac{1}{b}$, Solve: $2x \oplus 3 = x \oplus 12$.

- a) $x = 9$ b) $x = 6$ c) $x = 2$ d) $x = 18$ e) $x = 4\frac{1}{2}$

26. Given the following program determine the value that will be printed:

```
1   LET x = 0
2   LET y = 10
3   ADD 0.2*y to x and store resulting sum in x
4   REDUCE y by 1
5   IF x < y GOTO STEP 3
6   ELSE Print x
```

- a) 5.4 b) 6.8 c) 7.4 d) 8 e) 9.8

27. This year Matilda is as old as the sum of the ages of Carol and Fred. Two years ago she was three times as old as Carol. If Fred is five years older than Carol, How old is Matilda?

- a) 23 b) 9 c) 14 d) 13 e) 43

28. Let $f(x) = \sqrt{x^2 + c} - x$ where $c > 0$. For what value of x is $f(x) = c$?

- a) $x = c$ b) $x = c^2 + c$ c) $x = \frac{-1 \pm \sqrt{1 - 4c}}{2}$ d) $x = \frac{1 - c}{2}$ e) $f(x)$ can never equal c

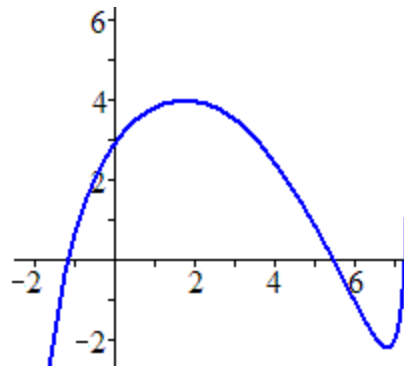
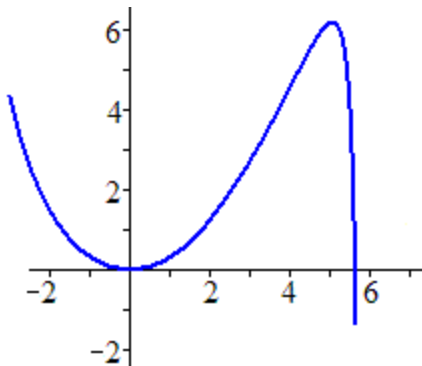
29. Find the equation of a line that is perpendicular to the line $4x - 3y = 12$ and that intersects it at $x = 6$.

- a) $y = 0.75x - 0.5$ b) $y = \frac{4}{3}x - 4$ c) $y = \frac{4}{3}x + 4$
 d) $y = -0.75x + 16.5$ e) $y = -0.75x + 8.5$

30. If $z = \frac{2y \cdot x}{x^2 + y^2}$ solve the expression for x .

- a) $x = \frac{y(2 - z)}{z}$ b) $x = \frac{y \pm (y - yz)}{z}$ c) $x = \frac{2y \pm \sqrt{y^2 - 4}}{2}$
 d) $x = y \pm 2\sqrt{y^2 - 1}$ e) $x = \frac{y \pm y\sqrt{1 - z^2}}{z}$

31. The left graph shows the function $f(x)$. Which functional expression best describes the right graph?



- a) $f(2 - x) + 4$ b) $4 - f(x + 2)$ c) $4 - f(2 - x)$ d) $4 - f(x - 2)$ e) $4 + f(x + 2)$

32. Sheryl has \$5 worth of silver coins (nickels, dimes, and quarters) in her coin jar. The total number of nickels is the same as the total number of dimes. What is the maximum number of coins in the jar.

- a) 20 b) 55 c) 62 d) 75 e) 100

33. Find the point of intersection farthest away from the origin of the following two functions:

$$f(x) = x - 3 \quad \text{and} \quad g(x) = \frac{x + 3}{x}$$

- a) $(2 + \sqrt{7}, -1 + \sqrt{7})$ b) $(2 - \sqrt{7}, -1 - \sqrt{7})$ c) (3, 2)
d) $(3 + \sqrt{6}, \sqrt{6})$ e) (-1, -4)

34. Given $f(x) = \sqrt{2x - 1}$ determine x where $f(x - 2) = 5$.

- a) 5 b) 13 c) 15 d) 3 e) None of these

35. Given the following information: Of the 135,000 voters 56% were women. 52% of the women and 47.5% of the men voted for the Democrat. How many votes did the Democrat get?

- a) 67,527 b) 71,145 c) 75,600 d) 75,222 e) None of these

36. For what values of m will the line $y = mx - 3$ be tangent to the parabola $y = x^2 + 2$.

- a) $m = \pm 2$ b) $m = \pm 5$ c) $m = \pm \sqrt{20}$ d) $m = \frac{\pm \sqrt{5}}{2}$ e) $m = \frac{3 \pm \sqrt{5}}{2}$

37. Find an expression for x in terms of C (but without using y), given the following two equations:

$$C = 2x + 6y - 10 \quad \text{and} \quad \frac{12}{x + 2y} = 1$$

- a) $x = \frac{11 + C}{2}$ b) $x = 26 - C$ c) $x = \frac{46 + C}{5}$ d) $x = \frac{46 - C}{5}$ e) $x = \frac{36 - C}{2}$

38. Jan has a working digital clock that shows international time (hours range from 0 to 23). He looks at his clock and notices that the hours are triple the minutes. Less than an hour earlier the minutes were the triple of the hours. If it is past 5:00 PM (17:00) how much time elapsed?

- a) 55 minutes b) 45 minutes c) 33 minutes d) 27 minutes e) 15 minutes

39. Solve for x : $\sqrt{2x^2 - 3} = 2x - 3$.

- a) $x = \frac{3 - \sqrt{3}}{2 - \sqrt{2}}$ b) $x = 3 + \sqrt{3}$ c) $x = 3 \pm \sqrt{6}$ d) $x = \frac{3 \pm \sqrt{3}}{2}$ e) $x = 3$

40. Which of the following functions has these two criteria: $f(0) = 0$ and $f(x + 1) = 2f(x) + 1$?

- a) $f(x) = 1 - 2^x$ b) $f(x) = 2x$ c) $f(x) = 2x^2 - x$
d) $f(x) = 2^x - 1$ e) $f(x) = -2^x - 1$

Algebra I - Answers

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|-----|---|-----|---|
| 1. | C | 21. | B |
| 2. | D | 22. | C |
| 3. | B | 23. | D |
| 4. | A | 24. | E |
| 5. | E | 25. | C |
| 6. | B | 26. | B |
| 7. | A | 27. | A |
| 8. | C | 28. | D |
| 9. | E | 29. | E |
| 10. | B | 30. | E |
| 11. | C | 31. | D |
| 12. | B | 32. | C |
| 13. | E | 33. | A |
| 14. | E | 34. | C |
| 15. | A | 35. | A |
| 16. | D | 36. | C |
| 17. | E | 37. | B |
| 18. | A | 38. | E |
| 19. | A | 39. | B |
| 20. | C | 40. | D |

Best of Three: 10, 20, 30

Sudden Death.: 5, 15, 25, 35, 40