Combat Models Final Investigation

In this investigation we will contrast and compare three models of combat.

- The first is called the conventional or direct fire model. This model describes historic battles like those fought in the Napoleonic and Civil Wars. Armies or navies lined up opposite each other and fired until one retreated and the other was victorious. The effectiveness of that fire and the number of combatants in each group will determine the outcome of the battle.
- The second model is a guerrilla combat model. This model describes situations like battles fought by guerrilla groups in Lebanon. Soldiers can’t see their opponents, but know that they are hiding in a region (perhaps of a city or in the jungle). They fire indiscriminately into the region hoping to hit their opponents. The larger the force in the region, the more likely they are to get hit, so a guerrilla combat unit is generally quite small.
- The third model pits a force using conventional tactics against a force using guerrilla tactics. This is called a combined combat model. One group is marching down the road and the other is hiding in the bushes. This describes the colonists vs England in the Revolutionary War, the Vietnam War, and some aspects of the wars in Afghanistan and Iraq.

1. We will develop differential equations that describe each situation.
2. We will set up Euler’s Method equations to investigate the situation, change parameters, and try to figure out what is going on.
3. We will solve the differential equations analytically using calculus. We will use the solutions to answer questions about each scenario. You might want to do some historical research in this project to support or enhance your mathematical findings.

Conventional Combat

In the conventional model, the rate at which one force loses its fighters is proportional to the number of opponents. The constant of proportionality is a measure of the quality of armament and the level of ability of the opposing force. Suppose we have force A fighting against force B.

1. Suppose a force of 3000 engages a force of 4000 in conventional battle and each has a proportionality constant of 0.04. The engagement is called off when either force reaches 1000. Use Euler’s Method to determine how many soldiers remain on the winning side. Suppose a force of 3500 engages a force of 4000 with the same proportionality constant. How many will remain on the winning side? 2000 against 4000?

2. Suppose A has 3000 troops and B has 4000 in conventional battle. If the soldiers in A are “twice as good” as those in B and the engagement is called off when either force reaches 1000, does A win? Suppose a definition of a “fair fight” is that each side reaches zero simultaneously. For the scenario in #2, how much better does A have to be for it to be a “fair fight”?

3. If the coefficient of effectiveness for Force A is $a$ and that Force B is $b$, what conditions are required for this to be a “fair fight”? If $a = 3b$, how many forces must B have to combat a full company of A?
Analytic Solution: Using the Chain Rule write a differential equation for $\frac{dB}{dA}$ in terms of $a$, $b$, $A$, and $B$ and solve the DE using analytic methods. Use the initial conditions $A(0) = A_0$ and $B(0) = B_0$. Compare your analytic solution to the numerical solution obtained in #1. Include a graph in your comparison.

Using your analytic solution, answer the questions in #2 and #3 above. Your solution should involve an expression relating the initial conditions and the coefficients of effectiveness. Compare these general results to your numerical solutions in #2 and #3.

4. At Iwo Jima in WWII, the Japanese had 21,500 soldiers and the US had 73,000 soldiers. They engaged in conventional warfare, but the Japanese were fighting from reinforced entrenchments. The coefficient of effectiveness for the Japanese was 0.0544 while that of the US side was 0.0106 (based on data after the battle). If these are correct, use your analytic solution to determine which side should win?

Using your analytic solution, find the number of soldiers remaining on the winning side when the other side has only 1500 remaining? The actual battle ended with 1500 Japanese survivors and 54,314 US survivors; compare this to your findings.

Guerrilla Combat

In the guerrilla campaign, the rate at which one force loses its fighters is jointly proportional to the number of opponents and the number of its own soldier in the region of combat. The constant of proportionality is a measure of the quality of armament and the level of ability of the opposing force and of the style of combat. It is typically between one hundredth and one thousandth of that for conventional combat (lots of shots are fired that miss their targets).

5. Suppose Force A of 30 engages Force B of 40 in guerrilla warfare and each has a constant of proportionality of 0.001. If the engagement is called off when only 10 remain, how many are left in Force B and the end of the battle?

In #6 and #7 use analytic methods similar to those used previously to answer the questions. Confirm your results numerically using the values given in #5.

6. Suppose the soldiers in A are “twice as good” as those in B, does A win? How much better does A have to be for it to be a “fair fight”?

7. If the coefficient of efficiency for Force A is $a$ and that Force B is $b$, what conditions are required for this to be a “fair fight”? If $a = 3b$, how many forces must B have to combat a full company of A?
Combined Combat

In a combined combat interaction, the rate at which the conventional force loses its fighters is proportional to the number of opponents while the rate at which the guerrilla force loses its fighters is jointly proportional to the number of opponents and the number of its own soldier in the region of combat. The constant of proportionalities are different, but each is a measure of the quality of armament, the level of ability of the opposing force, and the style of fighting employed.

8. Suppose a full platoon of Force A is searching for a squad of Force B in the jungle. Force B is setting up an ambush. When the battle starts, Force A is in full view of the defenders while Force B is hidden in jungle, so this is the combined format. Force A is the conventional force and Force B the guerrilla force. If the efficiency of Force A is 0.001 while that of Force B is 0.04 (due to the type of fighting employed), what is the outcome of the conflict?

In #9 use analytic methods similar to those used previously to answer the questions.

9. For this conflict, if the coefficient of efficiency for Force A is $a$ and that Force B is $b$, what conditions are required for this to be a “fair fight”? In Vietnam, the ratio of $a$ to $b$ was estimated as $\frac{a}{b} = \frac{1}{500}$. How large a Force A could a squad of Force B meet in a “fair fight”? How large a force must B have to combat a full company of A.

Extension

1. For the conventional warfare scenario, write and solve differential equations for $\frac{d}{dt}(A - B)$ and $\frac{d}{dt}(A + B)$, assuming $a = b$.

2. Use your results from question 1 to find closed form expressions for $A$ in terms of $t$ and $B$ in terms of $t$. Compare information about how $A$ and $B$ change over time that you get from these expressions to the information you have gotten numerically.
The basic building block of all Army organizations is the individual soldier. A small group of soldiers organized to maneuver and fire is called a squad. As elements of the Army's organizational structure become larger units, they contain more and more subordinate elements from combat arms, combat support and combat service support units.

A company is typically the smallest Army element to be given a designation and affiliation with higher headquarters at battalion and brigade level. This alphanumeric and branch designation causes an "element" to become a "unit."

**Squad** - 9 to 10 soldiers. Typically commanded by a sergeant or staff sergeant, a squad or section is the smallest element in the Army structure, and its size is dependent on its function.

**Platoon** - 16 to 44 soldiers. A platoon is led by a lieutenant with an NCO as second in command, and consists of two to four squads or sections.

**Company** - 62 to 190 soldiers. Three to five platoons form a company, which is commanded by a captain with a first sergeant as the commander's principle NCO assistant. An artillery unit of equivalent size is called a battery, and a comparable armored or air cavalry unit is called a troop.

**Battalion** - 300 to 1,000 soldiers. Four to six companies make up a battalion, which is normally commanded by a lieutenant colonel with a command sergeant major as principle NCO assistant. A battalion is capable of independent operations of limited duration and scope. An armored or air cavalry unit of equivalent size is called a squadron.

**Brigade** - 3,000 to 5,000 soldiers. A brigade headquarters commands the tactical operation of two to five organic or attached combat battalions. Normally commanded by a colonel with a command sergeant major as senior NCO, brigades are employed on independent or semi-independent operations. Armored cavalry, ranger and special forces units this size are categorized as regiments or groups.

**Division** - 10,000 to 15,000 soldiers. Usually consisting of three brigade-sized elements and commanded by a major general, divisions are numbered and assigned missions based on their structures. The division performs major tactical operations for the corps and can conduct sustained battles and engagements.

**Corps** - 20,000 to 45,000 soldiers. Two to five divisions constitute a corps, which is typically commanded by a lieutenant general. As the deployable level of command required to synchronize and sustain combat operations, the corps provides the framework for multi-national operations.

**Army** - 50,000 + soldiers. Typically commanded by a lieutenant general or higher, an army combines two or more corps. A theater army is the ranking Army component in a unified command, and it has operational and support responsibilities that are assigned by the theater commander in chief. The commander in chief and theater army commander may order formation of a field army to direct operations of assigned corps and divisions. An army group plans and directs campaigns in a theater, and is composed of two or more field armies under a designated commander. Army groups have not been employed by the Army since World War II.