Calculus Alternate Investigation
Exploring Pollution in the Great Lakes

In the Great Lakes in the United States, water flows from Lake Huron into Lake Erie, and from Lake Erie to Lake Ontario. From Lake Ontario, water flows through the St. Lawrence Seaway to the ocean. Each year, 11% of the water in Lake Huron flows into Lake Erie, while 36% of the water in Lake Erie flows into Lake Ontario, and 12% of the water in Lake Ontario flows out to the sea.

In years past, there were aluminum factories on each of the lakes, pumping a pollutant into the lakes. Last year, all of the plants closed down except for one plant on Lake Huron that continues to pump 25 tons of the pollutant into the lake each year. Presently, there are 3500, 1800, and 2400 tons of this pollutant dissolved uniformly in the water in the three lakes, respectively.

1. Write and solve differential equations that represent the change in pollution levels in each lake over time.

2. a. How many years will it take for the levels of pollutant in each of the three lakes to be reduced to 8% of their present levels? Which of the three lakes will reach this level first? Which will reach this level last? In the long run, what will the pollutant levels be in each lake?

b. If the final plant on Lake Huron were to close down at the end of 10 years, how long would it take for each of the three lakes to have less than 8% of their present levels? Carefully explain how you arrive at your answers.

c. Suppose the remaining plant had been located on Lake Erie instead of on Huron. Would this make any difference in the long-term level of pollution in Lake Ontario? Interpret your answer in the context of the problem and give a possible explanation for the outcome.

3. In answering the questions in 2, you could have used numerical solutions or analytic solutions to the differential equations. The differential equations for Lake Erie and Lake Ontario can be solved using integrating factors. If you solved the differential equations using numerical methods, now solve the D.E.'s using analytical methods. If you solved the D.E.'s using analytical methods, now solve the D.E.'s using numerical methods.

4. Discuss the differences in your analytic and numerical solutions.

5. Summarize the results of this investigation.