Predicting the Spread of an Infectious Disease

The world is being threatened by a new disease, SARS. The Center for Disease Control (CDC) offers the following information at http://www.cdc.gov/ncidod/sars/factsheet.htm:

**A new disease called SARS**
Severe acute respiratory syndrome (SARS) is a respiratory illness that has recently been reported in Asia, North America, and Europe. This fact sheet provides basic information about the disease and what is being done to combat its spread. To find out more about SARS, go to www.cdc.gov/ncidod/sars/ and www.who.int/csr/sars/en/. The Web sites are updated daily.

**Symptoms of SARS**
In general, SARS begins with a fever greater than 100.4°F [>38.0°C]. Other symptoms may include headache, an overall feeling of discomfort, and body aches. Some people also experience mild respiratory symptoms. After 2 to 7 days, SARS patients may develop a dry cough and have trouble breathing.

**How SARS spreads**
The primary way that SARS appears to spread is by close person-to-person contact. Most cases of SARS have involved people who cared for or lived with someone with SARS, or had direct contact with infectious material (for example, respiratory secretions) from a person who has SARS. Potential ways in which SARS can be spread include touching the skin of other people or objects that are contaminated with infectious droplets and then touching your eye(s), nose, or mouth. This can happen when someone who is sick with SARS coughs or sneezes droplets onto themselves, other people, or nearby surfaces. It also is possible that SARS can be spread more broadly through the air or by other ways that are currently not known.

**Possible cause of SARS**
Scientists at CDC and other laboratories have detected a previously unrecognized coronavirus in patients with SARS. The new coronavirus is the leading hypothesis for the cause of SARS.

In this investigation, we will develop a model for the spread of SARS in a closed population. The model is similar to those used to model the spread of other infectious diseases like measles, influenza, and AIDS. We are particularly interested in considering the effectiveness of isolation and quarantine of infected individuals, and wearing of masks by individuals concerned about “catching” SARS. We will consider what happens when a small group of people with an infectious disease is introduced into a larger, closed population? Does everyone in the larger population get the disease? Will the disease die out before it spreads? Is there some critical mass of infectious people that must exist for the disease to become an epidemic? When will the disease reach its maximum number, and what fraction of the total population has the disease at that time?

**Basic Assumptions**

We will assume that the disease has a very short incubation period, so that immediately after contacting the disease, the infected person can pass it on (there is a 10 day period in which the infected individual does not have symptoms, but can transmit the disease). We are also assuming in our model that once an individual recovers from the disease, that individual becomes permanently immune (or the disease results in death), and cannot catch it again. It is
not known at present if recovery gives immunity from SARS. These assumptions allow us to divide the population into four groups:

- the infected group \( (I) \) consisting of those individuals who presently have the disease and can transmit it to others.
- the susceptible group \( (S) \) consisting of those individuals who are not infected, but are capable of becoming infected.
- the recovered group \( (R) \) consisting of those who have had the disease and recovered and cannot become infected again
- and the terminal group that has died as a result of the disease \( (T) \).

The spread of the disease is governed by four rules:

1. The base population remains constant throughout the duration of the disease. We will neglect births, deaths due to factors unrelated to the disease, and immigration into or emigration out of the base population.

2. The rate of change of the susceptible population is proportional to the product of the number of members of the \( S \) and the number of members of \( I \). Call the constant of proportionality \( k \). The parameter \( k \) measures the transmission rate of the disease. Large value of \( k \) suggest a disease that is easily transmitted through personal interaction, while small values of \( k \) suggest a disease that is difficult to transmit through personal interaction. SARS is passed when an infected individual comes into contact with a susceptible individual. Not every contact results in transmission of the disease (thank goodness). The product of \( S \) and \( I \) is a measure of how possible contacts there are (this is similar to the predator-prey setting). There is a 10 day incubation period before symptoms show.

3. Individuals are removed from the infectious group at a rate proportional to the size of \( I \). Call the constant of proportionality \( r \). The value of the parameter \( r \) measures the rate of recovery (or death) from the disease. If a disease lasts a long time, \( r \) is small. If the disease is rapid, \( r \) is large.

4. Some of the individuals removed from the infectious group become Recovereds while others die from the disease and becomes Terminal. At the present time, the death rate from SARS is approximately 5%.

Use these 4 rules to define expressions for \( \frac{dS}{dt} \), \( \frac{dI}{dt} \), \( \frac{dR}{dt} \), and \( \frac{dT}{dt} \).

Computer (or Calculator) Investigation

Before doing any analysis, we want to set up and “play” with the model. We will use Euler's method to investigate how the values of \( k \), \( r \), \( S_0 \) and \( I_0 \) affect the dynamics of the system. Start with a simple system using \( N = 50,000 \), \( S_0 = 49,990 \), \( I_0 = 10 \), \( R_0 = T_0 = 0 \). This is our initial population for the investigation. We will later modify these values to see how the progress of the epidemic is altered by the change. At the present time, the research community has no good
value for $k$. For our investigation, we will assume initially that $k = 0.000005$, a value that is half of the rate often used for the spread of measles.

1. Write a system of 4 coupled differential equations describing $\frac{dS}{dt}$, $\frac{dI}{dt}$, $\frac{dR}{dt}$, and $\frac{dT}{dt}$ in terms of $I$, $S$, $R$, $T$, $k$, and $r$.

2. Set up the Euler iteration for this system of coupled differential equations. Start with a simple system using $N = 50,000$, $S_0 = 49,990$, $I_0 = 10$, $R_0 = T_0 = 0$, and $k = 0.000005$. Use $\Delta t = 1$ day for your iteration.

3. From Table 1, determine an approximate value for $r$ in your investigation. Be sure to include the 10 day incubation period before symptoms occur.

4. How does the progress of the disease change if $N = 40,000$, $N = 20,000$ or $N = 10,000$. For each of these populations, estimate the total number who eventually come down with SARS and the maximum number ill with SARS at one time.

5. With $N = 50,000$, suppose the disease is more difficult to catch than our model suggests. Use $k = 0.0000025$. How does this affect estimate the total number who come down with SARS and the maximum number ill with SARS at one time? Suppose the SARS virus is as easy to catch as measles, how does this affect estimate the total number who come down with SARS and the maximum number ill with SARS at one time?

6. At the present time, patients are treated with antibiotics, but no one knows if this treatment is effective. Using $N = 50,000$ and $k = 0.000005$ again, suppose a treatment is found that cuts the recovery time in half. How does this affect our estimate the total number who come down with SARS and the maximum number ill with SARS at one time?

7. Many people in areas where SARS is widespread try to reduce the spread of the disease by wearing masks. The Canadian Broadcast Corporation (CBC) did a report of the ability of masks to reduce the number of microscopic particles breathed in. Table 2 has some results of their study. Using $N = 50,000$ and $k = 0.000005$, if the value of $k$ is reduced by the suggested percentages, how would the progression of the disease be affected. If everyone wore Dust Masks, would that appreciably affect the disease? Suppose everyone wore Surgical Masks? N-95 masks?

**Model Extension (Optional):** Modify your model to include the possibility of re-infection after recovery. How would the model differ if you could be infected again immediately or if you had an immunity for a month?

*Analytic Investigation*
In our computer investigation using Euler’s Method, we have developed a number of conjectures about what would happened to the progress of the disease under different conditions. We want to verify those results and learn some crucial facts about epidemics by using Calculus.

1. When is \( \frac{dI}{dt} \) increasing and when is it decreasing? What does this mean about the spread of the disease. Using this result, explain why isolation and quarantine will be effective against SARS.

2. We have equations for \( \frac{dI}{dt} \) and \( \frac{dS}{dt} \). Use them to find \( \frac{dI}{dS} \). Solve this differential equation for \( I(S) \). Find \( I'(S) \) and \( I''(S) \) and use these derivatives to determine when the number of infected will begin to decrease. Does this match your solution from 1)? Explain any differences you see.

3. Use the function \( I(S) \) to determine the maximum number of individual ill at one time. What proportion of the population will be sick simultaneously? Do these results your graphs in the computer investigation?

4. How many Infectives will there be when the epidemic is over? Use this idea to determine the proportion of the population infected under the different situations considered in the computer investigation. How effective do the masks need to be to seriously affect the progress of the disease? Is it realistic to think isolation and wearing masks can work or must a vaccine be found?

Table 1: Severe Acute Respiratory Syndrome (SARS) in Singapore: Clinical Features of Index Patient and Initial Contacts

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This table presents information about the first 19 SARS patients in China. The numbers in the table are days after the onset of symptoms.

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aAbbreviations used in table: F, female; M, male; NR, not required; NS, not seen; Hosp, hospitalized.
bIndex patient.

Ahead of Print

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Ugis Bickis is an Environmental Hygienist who teaches at Queens University in Kingston. He’s been studying the effectiveness of masks for years. But he found a little surprise when he went shopping recently.

We bought N95s, the ones recommended for health care workers. Then we randomly picked a procedure mask (the kind you might see on your dentist), a surgical mask (like you’d see in a hospital) and one of those dust masks you often find in hardware stores, used for painting and other messy work. We also tossed in a bandana — a sampling of the products some Canadians have been using since the SARS scare began.

The test result is the “Filtration Efficiency”, it’s the percentage of sub micron particles that the mask filtered out. The higher the number, the greater number of particles are filtered out.

These values are averages from the article:

**Bandanna:** Folded Twice: 6 per cent of particles filtered out, Folded 4 times: 10 per cent

**Filter Mask:** 12 per cent

**Dust Mask:** 13 per cent

**Procedure Mask:** 32 per cent

**Surgical Mask:** 62 per cent

**N-95:** 98 per cent