A Model of Traffic Flow **Name _____**
Calculus Investigation

Everyone has at some time been on a multi-lane highway and encountered road construction that required the traffic to occupy only one lane each way. Naturally, the Department of Transportation would like to maximize the flow of traffic through this stretch of highway. What speed limit would be set for such a stretch of road to ensure the greatest traffic flow while also maintaining safety?

1. Develop a mathematical model to describe traffic flow.

2. Three different "rules of thumb" are commonly used to determine a safe flowing distance. Usually, the faster you are going the greater the distance must be between you and the car in front of you to give you time and distance to safely stop if the car in front of you stops suddenly. Change your model of traffic flow to accommodate each "rule of thumb."

    a. Follow 2 car lengths for every 10 mph.

    b. Follow two seconds behind the car in front.

    c. The distance needed to stop varies at different speeds and includes thinking and braking distances.

    Table 1 is from a driver's handbook and gives some approximate figures.

Speed (mph)	Thinking Distance (ft)	Braking Distance (ft)
20	20	20
30	30	45
40	40	80
50	50	125
60	60	180

3. Determine the speed that maximizes the flow of traffic for each of the three models in terms of the car length L.

4. Select the model that best describes traffic flow and has a maximum. Since this model is dependent on the length of the vehicle, the maximum traffic flow will also be dependent on the length. Graph traffic flow vs. speed and identify the ordered pair that represents maximum traffic flow for each of the following vehicles:
    a. Mini Cooper has length of 142.8 inches.
    b. Toyota Corolla has length of 178.3 inches.
    c. Hummer H2 has length of 189.9 inches.
    d. An eighteen-wheeler with average length between 70 and 80 feet

Adapted From: Stone, Alan, and Ian Huntley, "Easing the Traffic Jam," *Solving Real World Problems with Mathematics*, Volume 2, The Spode Group, Cranfield Press, Cranfield, UK, 1982.

EXTENSIONS:

5. The web site http://www.thetruckersreport.com/truck_facts.shtml#Length%20of%20eighteen%20wheelers gives the statement that "The length of time to stop an eighteen wheeler is 40% greater than that of an automobile." Apply this information to find the ordered pair that represents the speed for maximum traffic flow for an eighteen-wheeler.

6. Our experience suggests that drivers do not leave as much space between cars as they should. Let p be the fraction of the required stopping distance that the drivers actually leave between cars. Assume $0.1 \le p \le 1$. Find the optimal speed and its corresponding traffic flow that maximizes traffic flow in terms of L and p.
    a. Using the length of the Toyota Corolla with several different values of p to calculate the optimal speed.
    b. Use the length of the eighteen-wheeler and the issue of needing 40% greater stopping distance with several values of p to calculate optimal speed.

7. Some might argue that the drivers would never drive so close that they would not have sufficient time to react. These divers might (unconsciously, of course) leave all of the reaction distance but only a fraction of the braking distance. Adjust your model by multiplying braking distance by p, where $0.1 \le p \le 1$ to find the optimal speed and its corresponding traffic flow for the Corolla.

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