Calculus Investigation: A Subway Design Problem

Suppose that you were building a new subway system. How far apart would you place the stations in order to minimize the time that a typical subway user takes to get from his point of origin to his final destination? Creating a model that will allow you to answer that question is the purpose of this investigation. Consider these two questions before you begin creating your model.

- What is/are the advantage(s) (in terms of a commute of shorter duration) of having subway stations close together?
- What is/are the advantage(s) (in terms of a commute of shorter duration) of having subway stations far apart?

Since this is an optimization problem, you would like to have one variable (the dependent one) that is to be optimized, and another (the independent one) that is under your control.

Exercises for Investigation

1. What is the dependent variable in your problem? What is the independent variable in your problem?

2. All other quantities should be *constants*. (For example, the speed of the train.) Make a list of as many such quantities as you can think of and give them letter names. (For example, "let $r_{train} =$ the speed of the train.")

3. Draw and label axes with the independent variable on the horizontal axis and the dependent variable on the vertical axis. Make a rough sketch (no units are required) of what you think the graph should look like, showing the relationship between these two variables.

4. Identify the components that contribute to the time a person spends getting from point A to point B when he takes the subway. Note any simplifying assumptions you make.

5. Consider all of the components that determine the total travel time. Some of them may be assumed to be constants, while others will clearly depend upon the value of the independent variable. For each quantity that depends upon the independent variable, write an equation expressing that relationship. Then write a function in which the dependent variable is expressed in terms of only the independent variable and constants.

6. Use your calculator to graph your function. Does it resemble the graph you sketched for #3? Here are some observations made by one resident of New York City to help you determine values for some constants.

- It takes 45 seconds for him to walk one city block.
- He waits an average of 7 minutes for a subway train to arrive after he gets to a station. (However, this depends greatly upon the route and the hour of the day.)
- The stations in New York are separated by anywhere from 4 to 13 blocks, and they average about 10 blocks apart.
- A typical trip for this person is from his home near 235th Street to a movie theater on 68th street, a distance of about 167 blocks.
- The trains travel at a top speed of about 11 blocks per minute.
- An express train that follows the same route as a local train but skips certain stops, gains about 0.9 minutes for each stop it skips. (This time is greatly increased during rush hour, however, when passenger traffic is heavy.)

If there are other values that you need for your model, make reasonable guesses. Then plot your graph and **use calculus** to find the value of the independent variable that optimizes the dependent variable.

7. You are ready to explore the robustness of the model with respect to some of the assumptions. In particular, you will determine how the optimal separation of stations depends upon the values of the constants in your function. Return to the function with the constants expressed as letters (rather than with numerical values). Use calculus to find the optimal separation of subway stations. It will depend upon the value of some constants.

8. For each constant appearing in the solution, decide whether the relationship between the constant and the optimal solution seems reasonable. In particular, does increasing the value of the constant cause the optimal separation of stations to increase or decrease? Explain why these relationships make sense.

9. Many of the values that you assumed were constant are of course not constant. They vary, for example, by passenger and by time of day. Come up with a range of reasonable values for each of the constants and explore what range of values they produce for the optimal solution.