CALCULUS III EXAM REVIEW　　　　　Name: ____________________

Wednesday 6/4/08 1:30-3:30 pm　　　*There will be a calculator part and a non-calculator part.*

Disclaimer: This is designed to **aid** you in preparing for the exam but should not be regarded as sufficient. *Study your notes and homework, the textbook, and old tests.*

1. Evaluate each integral. Do not rely on your calculator.

   a. $\int_0^{\pi} \cos^2 x dx$　　b. $\int x\sqrt{1+kx}\, dx$　　c. $\int_0^1 \frac{1}{x^2-4}\, dx$　　d. $\int_1^{\infty} \frac{\ln x}{x} dx$

   e. $\int x \ln x\, dx$　　f. $\int e^{2x} \sin x\, dx$　　g. $\int t^2 e^{k\cdot t}\, dt$　　h. $\int_3^{\infty} \frac{dx}{x^2}$　　i. $\int_0^4 \frac{1}{(x-1)^2} dx$

2. Use the sixth degree Taylor polynomial for $f(x) = \sin\left(\frac{x^2}{2}\right)$ at $x=0$ to approximate $\int_0^1 \sin\left(\frac{x^2}{2}\right) dx$. (Use a memorized Taylor series, or redevelop that too for practice.)

3. The region in the plane bounded by $y = \frac{1}{x}$, $x=1$, and the x-axis is rotated around the x-axis. Find the volume of the resulting solid.

4. The base of a solid is the region in the x-y plane enclosed by the functions $y = x^2$ and $y = 4$. Slicing the solid perpendicular to the y-axis produces cross-sections that are squares. Find the volume of the solid.

5. Calculate the length of the graph of $y = \sqrt{x}$ from $x=0$ to $x=1$ and then from $x=9$ to $x=25$. Compare each to the length of the line segment joining the endpoints of each interval.

6. Determine whether each series converges or diverges. Support your answer with a named test for convergence and clear indication of how the test applies.

   a. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$　　b. $\sum_{n=0}^{\infty} (-1)^n \frac{n^2}{5^{n+1}}$　　c. $\sum_{k=0}^{\infty} \frac{2}{k(\ln k)^3}$　　d. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$　　e. $\sum_{n=0}^{\infty} \frac{3^{2n}}{(n+2)!}$

7. Find the interval of convergence for each power series.

   a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$　　b. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n (x+3)^n}{n!}$　　c. $\sum_{n=1}^{\infty} \frac{n(x-4)^n}{3^n}$　　d. $\sum_{n=0}^{\infty} (-1)^n 3^n (x+5)^n$

8. Find the slope of the tangent line to the three-leaf rose $r = \cos(3\theta)$ at $\theta = \frac{\pi}{6}$ and write the equation of the tangent line to the curve at $\theta = \frac{\pi}{6}$.

9. Let $f(x) = e^{-3x}$. Write the Taylor series representation for $f(x)$ about $x=0$; use sigma notation. For what values of x does this series converge?

10. Let $P(t) = 24 + 9t\, e^{(-t/3)}$ be a model for the temperature of the water in a pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find the average temperature of the water in the pond, in degrees Celsius, over the time interval [0,18] days.

11. Each of these expressions contains the integral symbol.

a) $\int f(x)dx$ b) $\int_1^3 f(x)dx$ c) $\int_0^x f(t)dt$, where x is the independent variable

i. Briefly explain the meaning of each expression. Using the word *integral* in your answer without explanation is not sufficient.

ii. Which of the above is equal to $\lim_{\Delta x \to 0} \sum_{k=0}^{n-1} f(x_k)\Delta x$, and under what conditions?

12. a. A train is moving away from a train station, beginning at time $t = 0$ seconds. The distance the train has traveled from the station is *D* feet. A reasonable model for data points $\left(t, \frac{\Delta D}{\Delta t}\right)$ over the first minute is linear, with slope equal to 0.6 and intercept 0. Write the differential equation modeling $\frac{dD}{dt}$ as a function of time, then solve it, expressing *D* as a function of *t*.

b. A small town is growing due to births exceeding deaths, and also due to immigration. Let t be the time since the year 2000 measured in years, and let *P* be the population of the town. In the year 2000 ($t = 0$) the population was 1200. A plot of $\left(P, \frac{\Delta P}{\Delta t}\right)$ is quite linear, with slope equal to 0.05 and intercept equal to 100. Interpret the meanings of these two quantities. Then write the differential equation modeling $\frac{dP}{dt}$ as a function of *P* and solve it, expressing *P* as a function of *t*.

13. Given $F(x) = \int_3^{x^3} \sec^5(t)\, dt$, find $\frac{dF}{dx}$.

14. a. State the Mean Value Theorem.
b. Sketch a graph and illustrate the meaning of the theorem on the graph.
c. If $f(x) = x + \frac{1}{x}$, find *c* to satisfy the MVT on the interval $[2, 4]$.

15. A solid has as its base the region bounded by the *x*-axis, the *y*-axis, and the graph of $y = \sqrt{4 - x}$. Cross-sections perpendicular to the *x*-axis are isosceles right triangles with one leg in the *x*-*y* plane. Find the volume of the solid.

16. Help! The Afsluitdijk dike in Holland has sprung a leak! Data taken every hour suggest that the rate of leakage is modeled by $r(t) = 150t^2 + 100t + 5000$, where t measures the number of hours that have elapsed since the leak first occurred, and $r(t)$ is the rate of the leak, in gallons per hour, at time t.

a. According to the model, how much water will leak out in the first 24 hours?
b. According to the model, at what time will twice as much water have leaked out as leaked out in the first 24 hours?
c. Let $W(t)$ be the total amount of water that leaks out of the dike during the first t hours. Write $W(t)$ as an explicit function of t.

17. Find the area inside the limaçon $r = 3 + 2\cos\theta$ and outer circle $r = 2$.

18. The graph of $y = f(t)$ for $-4 \le t \le 4$ is shown here.

Let $G(x) = \int_{-2}^{x} f(t)dt$.

a. Find $G(0)$
b. Find $G'(1)$
c. Sketch the graph of $y = G(x)$.