CALCULUS III TOPICS EXAM REVIEW SPRING TRIMESTER 2007-2008
Exam: Tuesday 6/4/08 1:30-3:30 pm Name: ____________________________
The exam will include a calculator part and a non-calculator part.

Disclaimer: This is designed to **aid** you in preparing for the exam but should not be regarded as sufficient. *Study your notes and homework, the textbook, and old tests.*

1. Evaluate each integral. Do not rely on your calculator.
   a. $\int_0^{\pi} \cos^2 x dx$ b. $\int x\sqrt{1+kx}\ dx$ c. $\int t^2 e^{k\cdot t}\ dt$
   d. $\int x \ln x\ dx$ e. $\int e^{2x} \sin x\ dx$

2. Consider the area enclosed by $f(x) = (x-4)^3$ and $g(x) = 3x-12$.
   a. Rotate this area about the line $x = 4$ and calculate the resulting volume.
   b. Rotate this area about the line $y = 0$ and calculate the resulting volume.

3. Write the second degree Taylor polynomial for $f(x) = e^{-x^2}$ at $x = 0$. Use the polynomial to estimate $\dfrac{1}{\sqrt[4]{e}}$.

4. Use the sixth degree Taylor polynomial for $f(x) = \sin\left(\frac{1}{2}x^2\right)$ at $x = 0$ to approximate $\int_0^1 \sin\left(\frac{1}{2}x^2\right) dx$. (Use a memorized Taylor series, or redevelop that too for practice.)

5. The region in the plane bounded by $y = \dfrac{1}{x}$, $x = 1$, and the *x*-axis is rotated around the *x*-axis. Find the volume of the resulting solid.

6. The base of a solid is the region in the *x*-*y* plane enclosed by the functions $y = x^2$ and $y = 4$. Slicing the solid perpendicular to the *y*-axis produces cross-sections that are squares. Find the volume of the solid.

7. Calculate the length of the graph of $y = \sqrt{x}$ from $x = 0$ to $x = 1$ and then from $x = 9$ to $x = 25$. Compare each to the length of the line segment joining the endpoints of each interval. Comment.

8. Determine whether each series converges or diverges. You must support your answer with a named test for convergence.
   a. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^3 + n^2 + n + 1 + \cos n}$ b. $\displaystyle\sum_{n=0}^{\infty} \frac{7n^5 - 9n^3 + 11n}{\pi n^5 + 9n^4 + 13n^2}$ c. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ d) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{2n}}{(n+2)!}$

9. Find the interval of convergence for each power series.
   a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$ b. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^n (x+3)^n}{n!}$ c. $\displaystyle\sum_{n=1}^{\infty} \frac{n^n (x-4)^n}{3^n}$
   d. $\displaystyle\sum_{n=0}^{\infty} (-1)^n 3^n (x+5)^n$

10. Write the first five terms of the Taylor series for $f(x) = \sqrt{x}$ about $x = 1$.

11. Let $f(x) = e^{-2x}$. Write the Taylor series representation for $f(x)$ about $x = 0$; use sigma notation. For what values of *x* does this series converge?

12. Each of these expressions contains the integral symbol.

a) $\int f(x)dx$ b) $\int_1^3 f(x)dx$ c) $\int_0^x f(t)dt$, where x is the independent variable

i. Briefly explain the meaning of each expression. Using the word *integral* in your answer without explanation is not sufficient.

ii. Which of the above is equal to $\lim_{\Delta x\to 0}\sum_{k=0}^{n-1} f(x_k)\Delta x$, and under what conditions?

13. Write an equation for F given that $F(\pi)=e$ and $F'(x)=\tan\left(x^2\right)$.

14. Given $F(x)=\int_{3^x}^{x^3}\sec^5(t)\,dt$, find $\dfrac{dF}{dx}$.

15. a. State the Mean Value Theorem.

b. Sketch a graph and illustrate the meaning of the theorem on the graph.

c. If $f(x)=x+\dfrac{1}{x}$, find *c* to satisfy the MVT on the interval $[1,2]$.

16. A solid has as its base the region bounded by the *x*-axis, the *y*-axis, and the graph of $y=\sqrt{4-x}$. Cross-sections perpendicular to the *x*-axis are isosceles right triangles with one leg in the *x*-*y* plane. Find the volume of the solid.

17. At a fast food restaurant the number of minutes a customer spends waiting in line is exponentially distributed with PDF $f(x)=0.3e^{-0.3x}$. The management will give away a free lunch to any customer who has to wait in line more than 14 minutes.

a. Find the probability that the management has to give away a free lunch.

b. If the restaurant serves 10000 people in a week, how many free lunches will the management have to give away?

c. What is the average waiting time at this restaurant?

18. Among the students at Central Middle School, the number of years *x* between visits to the dentist follows the probability density function $p(x)=\begin{cases}\dfrac{3}{x^4} & x\ge 1\\ 0 & x<1\end{cases}$.

a. Verify that this function is a PDF.

b. What is the mean number of years between visits to the dentist for these students?

c. What is the median number of years between visits to the dentist for these students?

Interpret your answers to (a) and (b). What do these values tell you about how often these students visit the dentist? What is the significance of the fact that the mean and the median are different? What causes the larger of the two to be larger?

19. A real number *P* is chosen at random between -4 and 4; a real number *Q* is chosen at random between -2 and 2. Find the probability that the product of *P* and the square of *Q* is greater than 1.

20. Determine the average value of the function $h(t)=\sin(2t)e^{1-\cos(2t)}$ on the interval $[-\pi,\pi]$.

21. Determine the equation of the tangent line to $r=3+8\sin\theta$ at $\theta=\dfrac{\pi}{6}$.

22. Find the area enclosed by the inner loop of the graph of $r=4+8\sin\theta$.

23. Find the area of the region outside $r=4+2\cos\theta$ and inside $r=3$.

Consider this evidence that Calculus is everywhere: "Only by taking infinitesimally small units for observation (the differential of history, that is, the individual tendencies of man) and attaining to the art of integrating them (that is, finding the sum of these infinitesimals) can we hope to arrive at the laws of history." *Leo Tolstoy, War and Peace*