Precalculus and Modeling
Third Trimester Review Solutions – June 2008

1. Check by using your calculator or MathCad. Be sure to get a good window for viewing! Think about domains, exact values of x-intercepts and y-intercepts and vertical and horizontal asymptotes.

2a) $y = 2\sin(x - \frac{\pi}{4}) + 1$

2b) $y = \tan\left(\frac{x}{2}\right) - 1$, assuming that the apparent vertical asymptotes are at $\pm\pi$

2c) $y = k(x+1)^2(x+4)(x-2)$ Since the graph goes through (0,-2), we can solve for "k":

$-2 = k(1)^2(4)(-2) = -8k$

$k = \frac{1}{4}$

$y = .25(x+1)^2(x+4)(x-2)$

3a)

$\tan\left(\frac{x}{3}\right) - 1 = 0$

$\tan\left(\frac{x}{3}\right) = 1$

$\frac{x}{3} = \tan^{-1}(1)$

$\frac{x}{3} = \frac{\pi}{4} + k\pi$

$x = \frac{3\pi}{4} + 3k\pi$, k is an integer

3b) $\tan^2(x) - \sec(x) - 1 = 0$

Use the identity $\tan^2(x) + 1 = \sec^2(x)$ to make a substitution for $\tan^2(x)$:

$\sec^2(x) - 1 - \sec(x) - 1 = 0$

$\sec^2(x) - \sec(x) - 2 = 0$

$(\sec(x) + 1)(\sec(x) - 2) = 0$

$\sec(x) + 1 = 0$ or $\sec(x) - 2 = 0$

$\sec(x) = -1$ or $\sec(x) = 2$

$\cos(x) = -1$ or $\cos(x) = \frac{1}{2}$

so $x = (2k+1)\pi$ or $x = \pm\frac{\pi}{3} + 2k\pi$, k is an integer

3c) $4\sin^2 x - 12\sin x - 7 = 0$

$(2\sin x - 7)(2\sin x + 1) = 0$

$2\sin x - 7 = 0$ or $2\sin x + 1 = 0$

$\sin x = \frac{7}{2}$ or $\sin x = -\frac{1}{2}$

no solution $\quad x = \sin^{-1}\left(-\frac{1}{2}\right)$

$x = \frac{7\pi}{6} + 2\pi k$ or $x = \frac{11\pi}{6} + 2\pi k$, k is an integer

3d) $3.5\cos\left(\frac{\pi}{6}(x+2)\right) + 4 = 5$

$3.5\cos\left(\frac{\pi}{6}(x+2)\right) = 1$

$\cos\left(\frac{\pi}{6}(x+2)\right) = \frac{1}{3.5}$

$\frac{\pi}{6}(x+2) = \cos^{-1}\left(\frac{1}{3.5}\right)$

Remember the symmetry of cosine.

$\frac{\pi}{6}(x+2) \approx 1.281 + 2k\pi$

$x + 2 \approx \frac{6}{\pi}(1.281 + 2k\pi)$

$x \approx .447 + 12k$, k is an integer

OR

$\frac{\pi}{6}(x+2) \approx -1.281 + 2k\pi$

$x + 2 \approx \frac{6}{\pi}(-1.281 + 2k\pi)$

$x \approx -4.447 + 12k$, k is an integer

3e) $\sin(2x) = \cos(x)$

$2\sin(x)\cos(x) = \cos(x)$

$2\sin(x)\cos(x) - \cos(x) = 0$

$\cos(x)[2\sin(x) - 1] = 0$

So, either $\cos(x) = 0$ or $\sin(x) = \frac{1}{2}$; therefore, $x = \frac{\pi}{2} + k\pi$ or $x = \frac{\pi}{6} + 2k\pi$ or $x = \frac{5\pi}{6} + 2k\pi$, k is an integer

3f) $(\sin 3x)(\cos 3x) = \sin^2 3x$

$(\sin 3x)(\cos 3x) - \sin^2 3x = 0$

$(\sin 3x)(\cos 3x - \sin 3x) = 0$

$\sin 3x = 0$ or $\cos 3x = \sin 3x$

$3x = k\pi$ or $\tan 3x = 1$

$x = \dfrac{k\pi}{3}$ or $3x = \dfrac{\pi}{4} + k\pi$

so $x = \dfrac{k\pi}{3}$ or $x = \dfrac{\pi}{12} + \dfrac{k\pi}{3}$, k is an integer

4 a) $p = \dfrac{1}{f}$ so $p = \dfrac{1}{60}$. To see two periods of the graph, use the equation from part (b) in your TI with an x-window of $[0, 1/30]$ and a y-window of $[-5, 5]$.

b) Amplitude = 5, no vertical shift, horizontal shift of .01 to the right so $y = 5\cos(120\pi(t - .01))$

c)

$3 = 5\cos(120\pi(t - .01))$

$\dfrac{3}{5} = \cos(120\pi(t - .01))$

$\cos^{-1}\left(\dfrac{3}{5}\right) = 120\pi(t - .01)$

$0.9273 = 120\pi(t - .01)$

$t \approx .0125$ seconds

5 a) $7^2 = 6^2 + 5^2 - 2(6)(5)\cos C$

$\cos C = 0.2 \Rightarrow C = \cos^{-1}(0.2) = 78.46^o$

b) $7^2 = 7^2 + 6^2 - 2(7)(6)\cos C$

$\cos C = .42857 \quad \Rightarrow \quad C = 64.62^o$ change is a decrease of 13.84^o

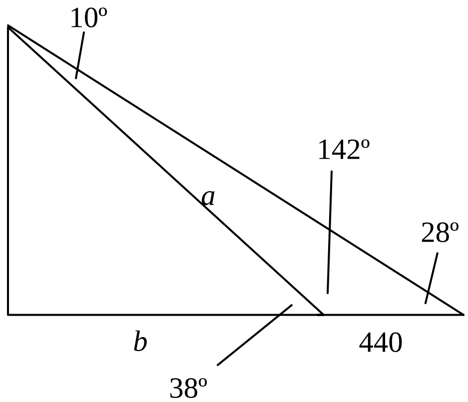
6 a) $\cos t = a \Rightarrow \sin t = -\sqrt{1 - a^2}$ (negative since we are in the 4th quadrant)

so $\tan t = \dfrac{-\sqrt{1 - a^2}}{a}$ (negative since we are in the 4th quadrant)

b) $\sin(t + \pi) = \sqrt{1 - a^2}$ since $t + \pi$ is in the 2nd quadrant

c) $\cos\left(t + \dfrac{\pi}{6}\right) = \cos t \cos\dfrac{\pi}{6} - \sin t \sin\dfrac{\pi}{6} = a \cdot \dfrac{\sqrt{3}}{2} - \left(-\sqrt{1 - a^2}\right) \cdot \dfrac{1}{2} = \dfrac{1}{2}\left(a\sqrt{3} + \sqrt{1 - a^2}\right)$

7.



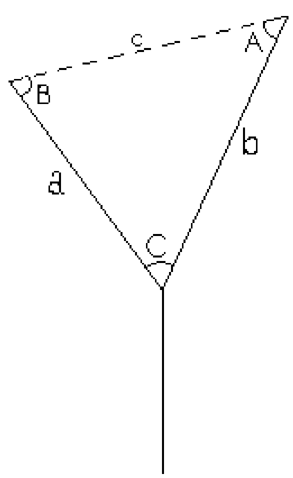
Using the Law of Sines: $\frac{a}{\sin(28^\circ)} = \frac{440}{\sin(10^\circ)}$ so $a = \frac{\sin(28^\circ)}{\sin(10^\circ)}(440) \approx 1189.57$ feet

Using right triangle trigonometry:

$$\cos(38^\circ) = \frac{b}{a}, \quad so \;\; b = a\cos(38^\circ) = 1189.57\cos(38^\circ) \approx 937.39 \;\; feet$$

So, the distance you are away from the building when you make your second observation is $937.39 + 440 = 1377.39$ feet.

8. You start out with your friend on the straight upward path. At 2:00 pm you take path *b*, and your friend takes path *a*. Angle C is 65º. At 2:30 pm you and your friend have reached the upper corners of the triangle, with a distance c between you. First find the lengths of sides a and b. Your friend spent half an hour traveling at 30 miles/hour, so



$$a = \frac{30 miles}{hour} * .5 hour = 15 miles$$

You spent half an hour traveling at 50 miles/hour, so

$$b = \frac{50 miles}{hour} * .5 hour = 25 miles$$

Fill in these values on your picture. The length of c gives you the distance between the two of you. Use the Law of Cosines!

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 15^2 + 25^2 - 2(15)(25)\cos(65^o)$$

$$c = \sqrt{15^2 + 25^2 - 2(15)(25)\cos(65^o)}$$

$$c \approx 23.088 miles$$

9. “3.8” tells you the average brightness of the star
“0.2” tells you the maximum variation from the average
“$\frac{\pi}{5}$” helps you determine that the period is 10 days

10. $y = a(x-1)^3 x(x+2)^2$ To find *a*, substitute (2,7) for *x* and *y*:

$7 = a(2-1)^3(2)(2+2)^2$

$7 = 32a$

$\frac{7}{32} = a$ **so** $y = \frac{7}{32}(x-1)^3 x(x+2)^2$

11. $y = \frac{3(x-1)(x+2)}{(x+1)(x-2)}$