

Calculus Challenge #1

Solution due: October 21, 2009

In your calculus class you have used the definition of derivative to develop the rules for differentiating the basic functions from precalculus. You used the definition to prove that

$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$ by multiplying the difference quotient $\frac{\sqrt{x+h} - \sqrt{x}}{h}$ by a clever form of 1 and simplifying before taking the limit.

Modify that technique to find the derivatives of the following functions *using the definition of derivative*.

a) $\frac{d}{dx} \sqrt[3]{x}$

b) $\frac{d}{dx} \sqrt[n]{x}$ for positive integer values of n .

c) Would the process used in b) work if n was a negative integer? Explain why or why not.

d) $\frac{d}{dx} \sqrt[5]{x^3}$

Try to generalize this last example to find $\frac{d}{dx} \sqrt[q]{x^p}$ using the definition of derivative and a clever choice of 1 when p and q are positive integers.