

Calculus Challenge #10

Solutions due March 24

Modeling the Unreasonable Effectiveness of Guerilla Warfare

This problem builds on Challenge Problem #14 and its solution from 2007-2008. Please review that problem before beginning your work.

http://courses.ncssm.edu/math/POW/POW07_08/Calculus%20Challenge%20%2314%20SOLUTION.pdf

As described in Problem 14 from 2007-2008, the defining equations for the direct fire combat model are very simple, $\frac{dA}{dt} = -bB$ and $\frac{dB}{dt} = -aA$. However, suppose one of the two combatants, we will call them G , is fighting a guerilla style battle while the other A is fighting a conventional battle. Since group A is still fighting a conventional directed fire battle, we have $\frac{dA}{dt} = -gG$. But what about group G ? How will their defining equation be altered by this restructuring of the conflict?

A guerilla style battle is also called an *area fire* (as opposed to *direct fire*) model. Instead of being exposed, the guerilla fighters are hidden. Their opponents can't see them to fire directly at them. All that is known is that the guerilla contingent is "over in that field". So, they fire into that area and hope for a hit. If a group of G combatants is fighting guerilla style, what is $\frac{dG}{dt}$?

Naturally, $\frac{dG}{dt}$ depends on the effectiveness and size of the opposing force as before, but what would improve the chances of the opposing force making a hit as it fires into the area holding the guerilla force? The larger the size of G , the more likely they will take a loss by a chance hit.

So, the change in the guerilla force over time can be modeled by $\frac{dG}{dt} = -aAG$.

In this setting, the coefficient of combat effectiveness a must also take into account the style of fighting, since the probability of a hit is dramatically reduced from the direct fire model. In general, the value of a is much smaller (by factors of 100 to 1000 depending on the terrain and type of weapons being used) than the coefficients in directed fire equations.

A battle between a force fighting conventional style and a force fighting guerilla style would be modeled by the coupled differential equations

$$\frac{dA}{dt} = -g \cdot G \quad \text{and} \quad \frac{dG}{dt} = -aAG.$$

1. Find A as a function of G by solving a differential equation involving $\frac{dA}{dG}$. Show that

$$A = \sqrt{2 \frac{g}{a} G + A_0^2 - 2 \frac{g}{a} G_0} \quad \text{and} \quad G = -\frac{aA^2}{2g} + \frac{aA_0^2}{2g} - G_0.$$

2. We can define a “fair fight” to be a battle in which the two forces reach zero at the same time. That is, the starting conditions for which the battle would end with $A = G = 0$. Use the constant in the equation from (1) to define a “fair fight”.

3. According to mathematician Courtney Coleman, during the Vietnam War, the ratio $\frac{a}{2g}$ was estimated at $\frac{1}{1000}$. Your result from (2) should indicate that a group of $G_0 = 10$ guerillas could be effective against a force 10 times their size fighting a conventional battle. Fill in the table below to illustrate how the effectiveness of guerilla warfare is reduced as the size of the guerilla force increases.

G_0	10	20	30	50	100	1000	1200
A_0	100						
Ratio	10						

Comparing the size of the Guerilla force G_0 with A_0 in a fair fight

4. We can find $A(t)$, but we have to be a bit clever to do it. We know that $G = -\frac{aA^2}{2g} + \frac{aA_0^2}{2g} - G_0$ and further, that $\frac{dA}{dt} = -gG$. Use these two equations to find $A(t)$. Does the sign of $\frac{aA_0^2}{2g} - G_0$ affect the method of integration and the final result? Explain why or why not.

5. Find $G(t)$.

6. The consequence of having small guerilla units ($G_0 = 10$) in Vietnam is that the total armed forces of the US and South Vietnamese combined needed to be about 10 times that of the Viet Cong and North Vietnamese for the US and South Vietnamese to have a reasonable chance of victory. In 1968, the ratio of forces was at 6 to 1. General Westmoreland asked for an additional 206,000 troops, but this request was rejected by President Johnson. Would the additional troops have turned the tide of the war? The table below gives the estimated number of forces in Vietnam at the time of the request.

Conventional Forces		Guerrilla Forces	
American	510,000	North Vietnamese	50,000
South Vietnamese: Regulars	600,000	Viet Cong	230,000
South Vietnamese: local defense	500,000		
Other Allies	70,000		
Total	1,680,000		280,000

Reference: Coleman, Courtney S. "Combat Models", in *Differential Equation Models*, Martin Braun, Courtney Coleman, and Donald Drew, Editors, Vol 1 of *Models in Applied Mathematics*, William Lucas, Editor, Springer-Verlag, NY, 1983.