Calculus Challenge Problem #4

Due December 9, 2009

Elasticity of Demand

If the revenue is given by

$$R(p) = p \cdot D(p)$$

and the elasticity of demand by

$$E(p) = \left(\frac{dD}{dp} \cdot\right) \frac{p}{D(p)}.$$

(1) Find $\frac{dR}{dp}$, the rate of change of revenue with respect to price, by differentiating $R(p) = p \cdot D(p)$. Show that $\frac{dR}{dp} = D(p)[1 + E(p)]$ after some simplification.

Since $R(p) = p \cdot D(p)$, $\frac{dR}{dp} = pD'(p) + D(p)$. Now, on the right hand side, factor out $D(p)$ so that $\frac{dR}{dp} = D(p)\left(p\frac{D'(p)}{D(p)} + 1\right)$. Since $E(p) = \left(\frac{dD}{dp} \cdot\right) \frac{p}{D(p)}$, we have $\frac{dR}{dp} = D(p)[E(p) + 1]$.

(2) There are three possibilities for the behavior of revenues at a price $p = p^*$ that follow from the equation $\frac{dR}{dp}$ in terms of D and E. (We are assuming that $D(p)$ has a negative slope, so demand decreases as price increases.)

a. When $-1 < E(p^*) < 0$, demand is said to be ***inelastic***. Describe what you know about R' and R in this case and what this means in context.

If $-1 < E(p^*) < 0$, then $\frac{dR}{dp} = D(p)[E(p) + 1]$ varies from 0 to 1. Then $\frac{dR}{dp} > 0$ which means an increase in price from p^* will result in an increase in revenue.

b. When $E(p) < -1$, demand is said to be ***elastic***. Describe what you know about R' and R in this case and what this means in context.

If $E(p^*) < -1$, then $\frac{dR}{dp} = D(p)[E(p)+1]$ must be negative. If $\frac{dR}{dp} < 0$, then any increase in price from p^* will result in a decrease in revenue.

c. When $E(p) = -1$, demand is said to be ***unit elastic***. Describe what you know about R' and R in this case and what this means in context.

If $E(p) = -1$, then $\frac{dR}{dp} = D(p)[E(p)+1] = 0$ Then $\frac{dR}{dp} = 0$. Any increase in price will be met with a decrease in demand that exactly offset. The revenue will not change regardless of price.

(3) Suppose the demand function for a graphing calculator is $D(p) = 1000 \cdot e^{-0.015p}$

If $D(p) = 1000 \cdot e^{-0.015p}$, and $E(p) = \left(\frac{dD}{dp} \cdot\right) \frac{p}{D(p)}$, then $E(p) = \frac{p\left(-150e^{-0.015p}\right)}{1000 \cdot e^{-0.015p}} = -0.015p$.

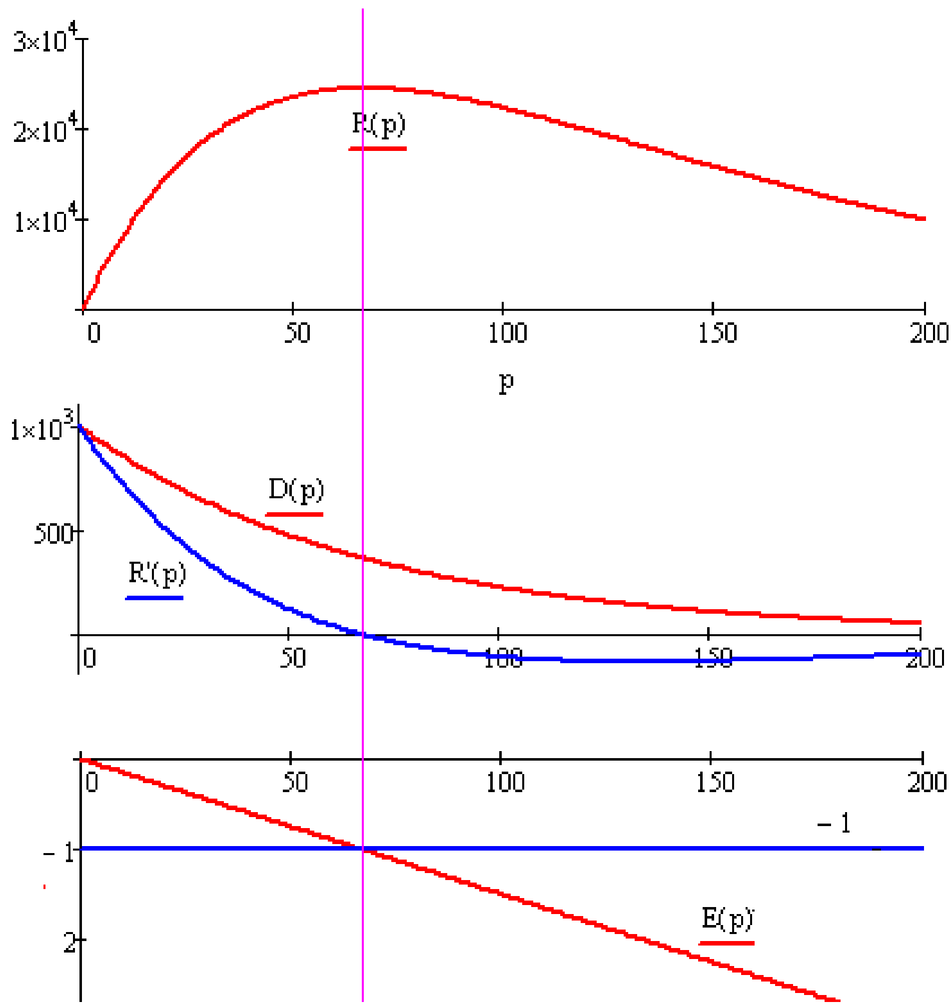
a. What is the elasticity of demand associated with a price of $100?

$E(100) = -0.015 \cdot (100) = -1.5$. The demand is elastic. The demand is $D(100) = 223$ creating a revenue of $22,300. Increasing the price to $101 will result in a demand of $D(101) = 220$ and a resulting revenue of $22,000. The increase in price resulted in a decrease in revenue, as expected.

b. What is the elasticity of demand associated with a price of $60?

$E(100) = -0.015 \cdot (60) = -0.9$. The demand is inelastic. The demand is $D(60) = 407$ creating a revenue of $24,420. Increasing the price to $61 will result in a demand of $D(61) = 401$ and a resulting revenue of $24,461. The increase in price resulted in an increase in revenue.

c. Graph *D(p), R(p), E(p),* and *R' (p).* Explain the relationships between the graphs of *D(p), R(p), E(p),* and *R' (p)*



As can be seen in the graphs above, $E(p) = -1$ at p = 66.67. This price also corresponds to the location of the maximum Revenue and the point at which the change in Revenue is zero.

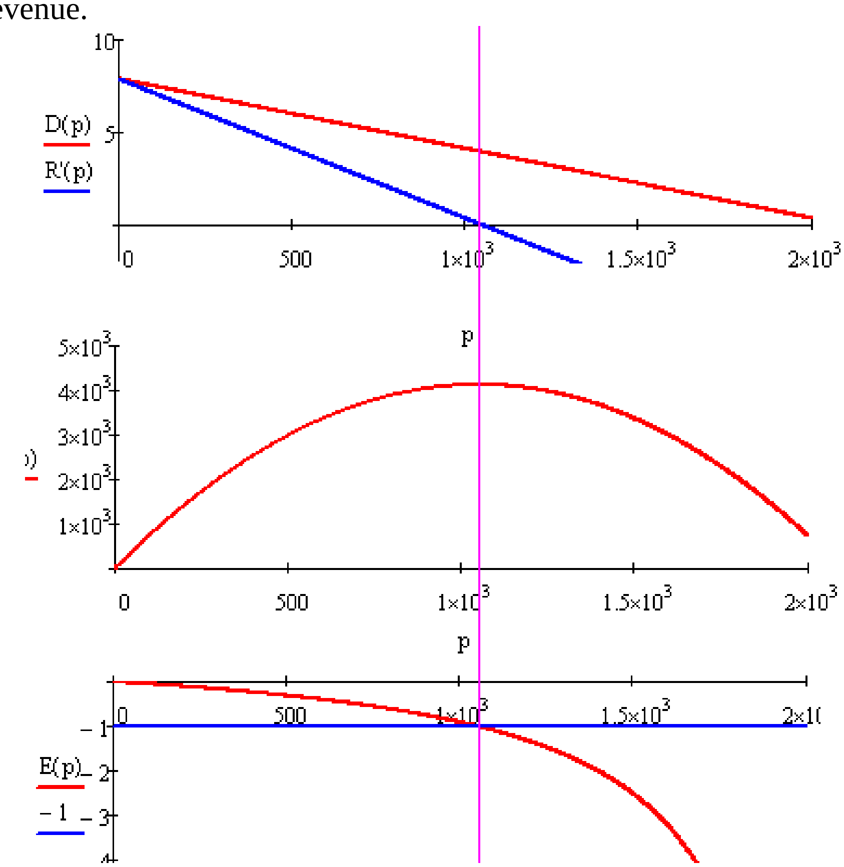
d. Investigate the change in revenue associated with each price to determine how the elasticity of demand is related to the way that revenue responds to price increases.

See responses in b) and c).

4) Terrence J. Wales described the demand for distilled spirits with the equation $q = -0.00375p + 7.87$ with p representing the retail price of liquor in dollars per case and q representing the average number of cases of distilled spirits purchased annually by individual consumers.

a. What is the function of elasticity in terms of price?

If $D(p) = -0.00375p + 7.87$, and $E(p) = \left(\frac{dD}{dp} \cdot \right)\frac{p}{D(p)}$, then $E(p) = \frac{p(-0.00375)}{-0.00375p + 7.87}$ which is equal to -1 at p = \$1049.33. For prices below this value E > -1, so the price is inelastic. Increases in price result in increased revenue. For prices above this price, increases in price will decrease the revenue.



b. What is the elasticity if liquor costs \$52.80 per case? What does this value mean?

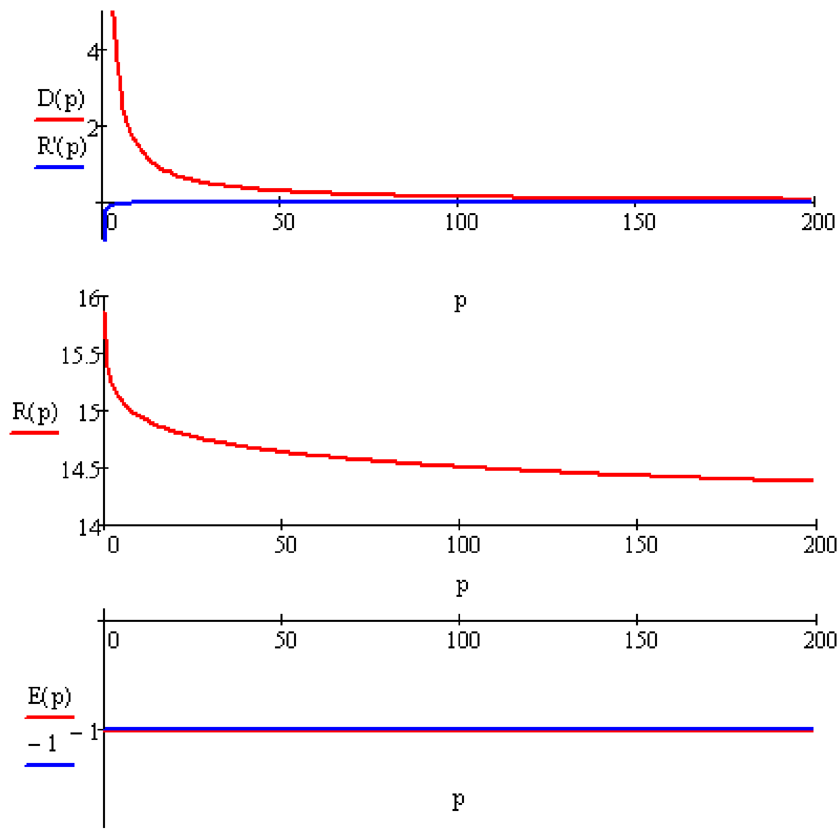
$$E(p) = \frac{(52.80)(-0.00375)}{-0.00375(52.80) + 7.87} = -0.026.$$ Increasing the price from \$52.80 will increase the revenue.

5) Nisbet and Vakil modeled the market for marijuana with the demand equation

$$D(p) = 15.4p^{-1.013}.$$

If the goal of the police is to decrease the revenue for drug dealers, law enforcement can either focus their attention on buyers (which decreases demand) or they can focus on dealers (which increases prices). Use the concept of elasticity to investigate how revenue would change as a result of focusing on dealers versus focusing on buyers.

Since $D(p) = 15.4p^{-1.013}$, $E(p) = \dfrac{p\left(15.5(-1.013)\,p^{-2.013}\right)}{15.4p^{-1.013}} = -1.013$. The elasticity of demand is constant and almost equal to -1. So, no matter what the price, the revenue will stay essentially constant.



If law enforcement focuses on dealers, the supply would decrease so the price would increase. The revenue would stay constant, so the amount purchased would be reduced.

If law enforcement focuses on buyers, the demand would increase and since the revenue stays the same, the amount purchased would increase as prices fell. To reduce the amount of marijuana purchased, the police should go after the dealers rather than the buyers.

(6) What demand function has an elasticity of -1 for all values of *p?*

If $E(p) = -1$, then $\frac{p \cdot D'(p)}{D(p)} = -1$, so we need a function with the property $D(p) = -p \cdot D'(p)$. What function is $-p$ times its derivative? Or $\frac{D(p)}{-p} = D'(p)$, what function divided by $-p$ is its own derivative? Thinking about the derivative rules we know, we find that $\frac{d}{dp}\left(\frac{1}{p}\right) = \frac{-1}{p^2}$, and $\frac{-1}{p^2} = \frac{\left(\frac{1}{p}\right)}{-p}$. More generally, we have $D(p) = \frac{k}{p}$ for any non-zero value of *k*. In this economic context, $k > 0$.

References:

Bartkovich, Kevin, et al, *Contemporary Calculus through Applications*, 166-170, 1996.

Wales, Terrence J.; "Distilled Spirits and Interstate Consumption Effects," *The American Economic Review* 57(4), 1968.

Nisbet, C. T. and F. Vakil, "Some Estimates of Price and Expenditure Elasticities of Demand for Marijuana among UCLA Students." *The Review of Economics and Statistics* 54(4), 1972.