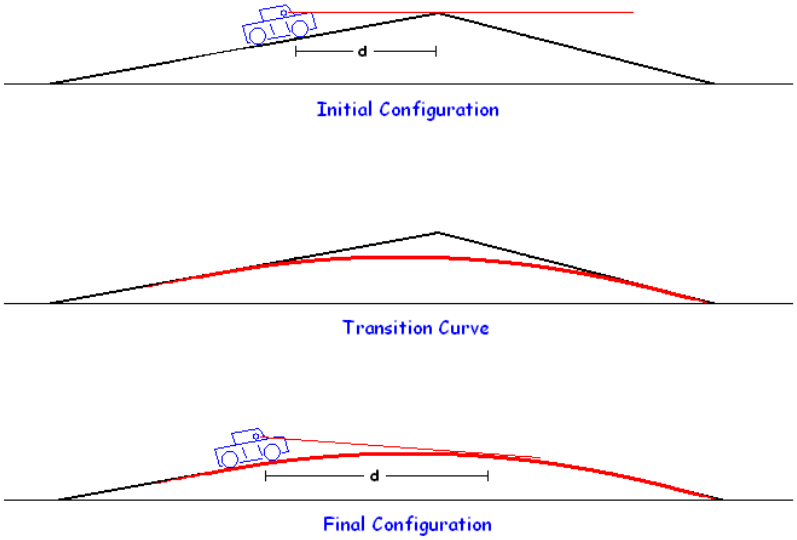
Calculus Challenge #3

Solutions Due Nov. 18

In this problem, we are interested in creating a transition curve. A transition curve is a curve in the plan which joins smoothly two line segments having different slopes (typically one positive and the other negative).

Consider the situation in designing a highway system. The highway proceeds from an uphill segment with slope S_1 to a downhill slope S_2. If no transition curve is cut, the visibility just before the top of the hill will be dangerously short.

Engineers want to join these two segments with a transition curve so that the horizon as seen from someone in the driver's seat *h* feet above the road surface is always at least *d* feet away.



a) Find a quadratic function that can be used as a transition curve to allow at least 250 feet of visibility down a road which joins an uphill slope of 3 degrees to a downhill slope of 3 degrees. Assume the driver's eye is 4 feet vertically above the road.

b) Use a circular sector to solve the problem posed in a).

c) Generalize your result. Find a curve that can be used to allow at least *d* feet of visibility down a road which joins an uphill slope of slope *S* to a downhill slope of *S*. Assume the driver's eye is *h* feet vertically above the road.

d) In a) we assumed the driver's eye was 4 feet vertically above the road. A more realistic model would have the driver's eye 4 feet along a perpendicular to the road. Use any method (quadratic, circle, or other function) to find a curve that can be used as a transition curve to allow at least 250 feet of visibility down a road which joins an uphill slope of 3 degrees to a downhill slope of 3 degrees. How much does this distinction affect the solution?

e) Find a curve that can be used to allow at least 250 feet of visibility down a road which joins an uphill slope of 2 degrees to a downhill slope of 3 degrees. Assume, as in a), the driver's eye is 4 feet vertically above the road.