

NCAAPMT Calculus Challenge 2010-2011

Challenge #1 Due: September 29, 2010

All answers must be supported.

Consider the recursively defined function $f(x) = \begin{cases} x-10 & \text{if } x > 100 \\ f(f(x+11)) & \text{if } x \leq 100 \end{cases}$.

- Find the values of $f(92)$, $f(85)$, $f(92.5)$ and $f(30\pi)$.
- Compute several more values of $f(x)$ for both integer and non-integer values of x . Once you are satisfied that you understand how the function works, sketch an anatomically correct graph on the domain of $[90, 110]$. Explain why the graph must look as you claim it does.
- By appealing to your graph or otherwise, find $f'(x)$ and clearly indicate the domain of the derivative.
- Let $g(x) = f(f(x))$ and let $h(x) = f(x-10)$. Find a relationship between the two functions g and h . Explain why you believe this is the correct relationship.
- Let $g(x) = f(f(x))$. Find $g'(x)$. Clearly indicate the domain of $g'(x)$.
- Find values of a , b , and c so that $F(x) = \begin{cases} x-a & \text{if } x \geq b \\ F(F(x+c)) & \text{if } x < b \end{cases}$ has the graph shown below:

