

### A Subway Design Problem

*In this week’s challenge, the primary focus is on developing an appropriate mathematical model. The calculus component of the problem should be reasonably straightforward once you have a function model that captures the important features of the situation.*

Suppose that you were building a new subway system. How far apart would you place the stations in order to minimize the time that a typical subway user takes to get from his point of origin to his final destination? Creating a model that will allow you to answer that question is the purpose of this challenge. Consider these two questions before you begin creating your model.

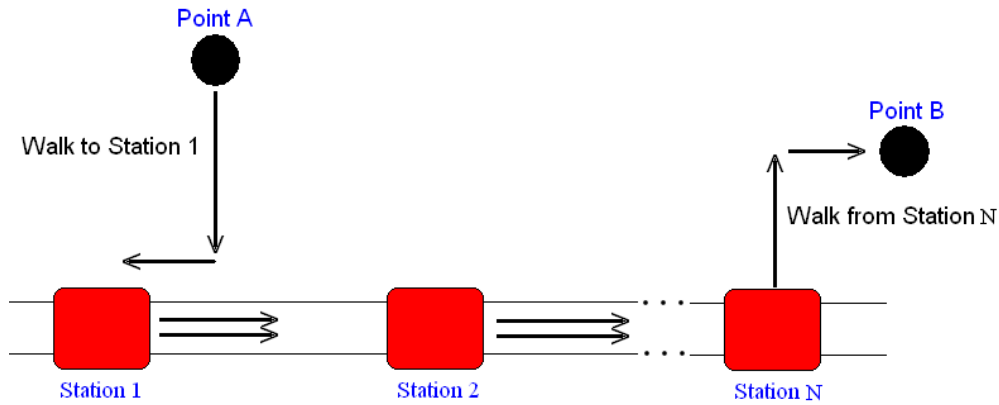
- What is/are the advantage(s) (in terms of a commute of shorter duration) of having subway stations close together?
- What is/are the advantage(s) (in terms of a commute of shorter duration) of having subway stations far apart?

Now, since this is an optimization problem, you would like to have one variable (the dependent one) that is to be optimized, and another (the independent one) that is under your control. Issues to consider:

- What is the dependent variable in your problem?
- What is the independent variable in your problem?
- We would like for all other quantities to be *constants*. (For example, the speed of the train.) Make a list of as many such quantities as you can think of and give them letters. (For example, “let  $S_{train}$  = the speed of the train.”) You will want to refer back to this list later.

Before you begin doing any math, draw and label axes with your independent variable on the horizontal axis and your dependent variable on the vertical axis. Make a rough sketch (no units are required) of what you think the graph should look like showing the relationship between these two variables.





Now begin making your mathematical model for the time it takes to travel between arbitrary points **A** and **B** in the city. Imagine that Points **A** and **B** are chosen at random, so the distance you must walk left or right varies from 0 to  $\frac{D}{2}$ , where  $D$  is the distance you choose between stations. You will probably have to make numerous simplifying assumptions along the way in order to keep the model simple enough to work with. That is natural. Make a note of what assumptions you make.

When your model feels complete, look at the quantities it involves. Some of them may be assumed to be constants, while others will clearly depend upon the value of the independent variable,  $D$ . Refine your model until it is as function of the single variable  $D$  and other constants and parameters.

1) Using your model, find the distance  $D$  between stations that will minimize the time needed to travel between random points **A** and **B**. Your solution will be in terms of the other constants and parameters used in the model.

2) Here are some observations made by one resident of New York City:

- It takes 45 seconds for me to walk one city block.
- Times vary at different times of day, but generally, trains arrive approximately every 15 minutes. With delays during the day, you should plan on a random arrival time that averages 15 minutes between arrivals.
- A typical trip for our resident is from his home near 235<sup>th</sup> Street to a movie theater on 68<sup>th</sup> street. This is 167 blocks.
- The trains travel at a top speed of about 11 blocks per minute.
- An express train that follows the same route as a local train but skips certain stops, gains about 0.9 minutes for each stop it skips.

Using these values and others that you estimate, what does your model say the distance between stations should be for New York City?

3) The stations in New York vary greatly, but average about 10 blocks apart. Does your model improve on this?