Calculus Challenge Problems #5

Solutions due December 1

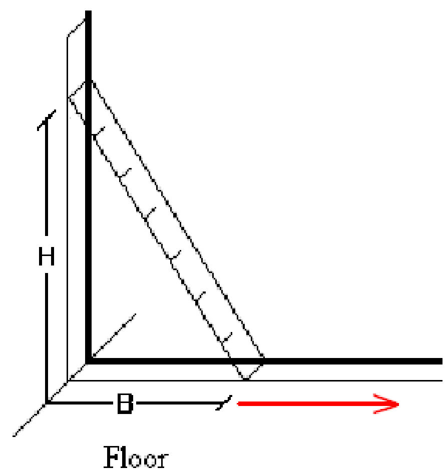
Related Rates

This week's challenge is more traditional than the modeling problem last week. It is a collection of related rates problems that don't quite fit anywhere else, so I thought I would put them together in a single challenge. Note that the solutions are due after Thanksgiving week, so you have three weeks to do this challenge.

1. A rectangle has two sides along the positive coordinate axes, and its upper right corner lies on the curve $x^3 - 2xy^2 + y^3 + 1 = 0$. How fast is the area of this rectangle changing as the point passes the position (2, 3) if it is moving so that $\frac{dx}{dt} = 1$ unit per second?

2. By now I'm sure you have done the problem of a ladder sliding down the wall. Just in case you haven't done this problem yet, I'll give you the opportunity now.

If a meter stick slides down a wall so that the bottom of the meter stick moves away from the wall at a constant speed of 5 cm/sec, according to your model, how far above the ground is the top of the meter stick when it breaks the sound barrier?



3. **The Extension Ladder Extension**

One nice extension to the classic ladder problem is to consider an extension ladder of length *L* that can contract and elongate (it is an extension ladder so it can do this) as it slides down the wall. The ladder starts out flat along the wall and the foot of the ladder is moving away from the ladder at a constant speed, *s*. Now suppose the ladder moves down the wall in such a way that *the distance the top moves down the wall is equal to the distance the bottom has moved away from the wall*. In order for this to happen, the ladder needs to extend and contract as it slides down the wall.

As the ladder slides to the ground,

a) When is the ladder contracting and when is it extending?

b) Where is the ladder the longest and where is it the shortest?

4. A thunderstorm is dropping rain at the rate of *k* inches per hour into a conical tank of water of radius *R* feet and height *H* feet. Let $\frac{dh}{dt}$ represent the rate at which the depth of the water in the cone is increasing. Show that the ratio of $\frac{dh}{dt}$ to the rate of rainfall is equal to the ratio between the area of the tank's opening to the area of the water surface.