

NCAAPMT Calculus Challenge 2011-2012

Challenge #2

Due: October 12, 2011

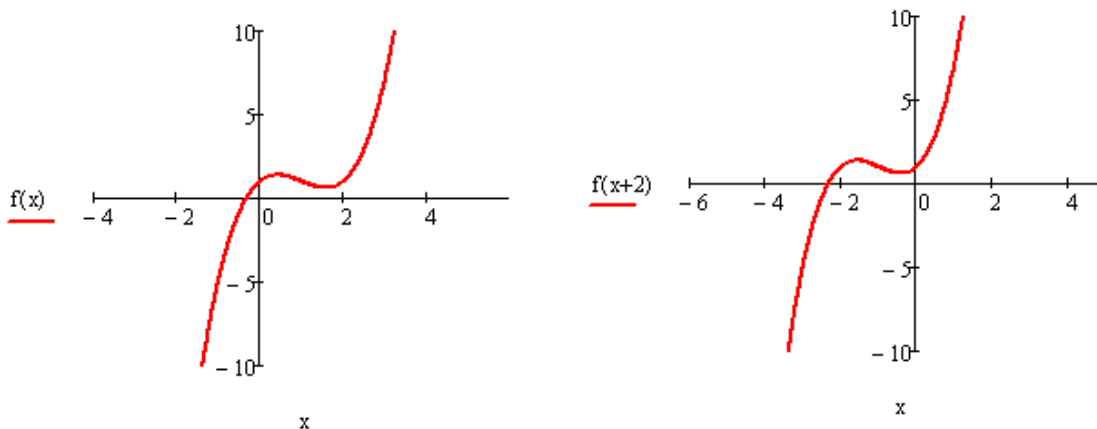
Who Needs Calculus for Derivatives?

One of the first things you learn in Calculus is how to find derivatives and to write the equation of a line tangent to a curve. In this challenge, we will try to convince you that you didn't need to learn calculus to do either.

1) First, use the methods of calculus to find the equation of the line tangent to $f(x) = ax^3 + bx^2 + cx + d$ at $x = 0$. Now, compare the equation of the tangent line to that of the polynomial. Is the result a surprise? Use calculus to prove that the line tangent to any polynomial at $x = 0$ is always the linear component of the polynomial.

2) Suppose we want to find the line tangent to $f(x) = 5x^3 - 3x^2 + x - 5$ at $x = 2$.

In 1) above, we saw that it is easy to find the tangent at $x = 0$. A horizontal shift doesn't change the shape of a curve, so would it work to shift the curve over so that the tangent we want is now at $x = 0$, use the method in 1), then shift back into its natural position? If this method works, explain why and write a general proof. If it doesn't always work, explain why not.



There is another way to find derivatives using the concept of the slope of a tangent line without resorting to the limit definition of derivative.

3) Let $f(x) = ax^2$. Suppose we want to know the derivative of f . We know that the derivative at any value of x will allow us to write the equation of a line tangent to the curve. This means that there is a slope M and a line $y = Mx + B$ that is tangent to

$f(x) = ax^2$. We want to know the value of M for which the equation $ax^2 = Mx + B$ has only one solution, since the curves are tangent and intersect at only one point. Use this idea to show that for $f(x) = ax^2$, $M = 2ax$ and consequently, that $\frac{d}{dx}(ax^2) = 2ax$.

4) Try the method of single point of intersection developed in 3) on $f(x) = ax^2 + bx + c$.

5) Now let $g(x) = \frac{a}{x}$. Use the method describe in 3) to show that if $\frac{a}{x} = Mx + B$ has only one solution, then $M = \frac{-a}{x^2}$ (this problem is more challenging than the last).

6) Use the method of single point of intersection on $h(x) = a\sqrt{x}$.

7) For what kinds of functions would the single solution to $f(x) = Mx + B$ approach fail?