

## NCAAPMT Calculus Challenge 2011-2012

### Challenge #4

Due: November 9, 2011

### A Variation on the Classic Wire Problem

You have all probably done “the wire problem”. Take a wire of length  $L$ , (a pipe cleaner works well), cut it at some point along its length. Use one section to make a square and the other section to make a circle. Find the point at which the wire should be cut to maximize the total area of the circle and square. Calculus teachers like to give this problem since the optimal location is at one of the endpoints of the domain. This reinforces the need to not only check critical values of the objective function, but also consider the endpoints of its domain.

We are going to look at the same problem, but instead focus on finding the location for the cut that *minimizes* the total area of the two figures.



- 1) How much wire should be used for the circle and how much for the square if we wish to *minimize* the total area of the two regions so defined? Prove that the area is minimal.
- 2) Now, with a pipe cleaner or a coat hanger, actually make the circle and square with dimensions given by your solution to 1). Put the two figures together (take a photo and submit it with you solution if you want). What is the relationship between these two figures? Prove that your observation is, in fact, correct.
- 3) Is this just a coincidence? Suppose instead of a circle and a square, you find the length of wire needed to bend the wire into a circle and an equilateral triangle which minimizes the combined area. Does the relationship found in 2) still exist? Prove this result.
- 4) Suppose instead you find the length of wire needed so that a square and an equilateral triangle are formed which minimize the combined area. Is the square inscribed in the triangle or the triangle inscribed in the square (or neither)? Prove this result.
- 5). Repeat 3) with a circle and either a pentagon, hexagon, or octagon. What did you find?
- 6) As a thought experiment (this means, don't actually do the computation), suppose you find the cut point so that a 7-gon and a 100-gon are created with minimum total area. If you created the polygons and put the two together, what would you see?

Question 7), below, is really hard and messy, so it will not be a part of the graded assignment.

**Super Challenge:** The area of a regular  $n$ -gon of side length  $s$  is  $A = \frac{s^2}{4} \cdot n \cdot \cot\left(\frac{\pi}{n}\right)$ . Prove that the  $n$ -gon and circle always have the relationship found in 2).