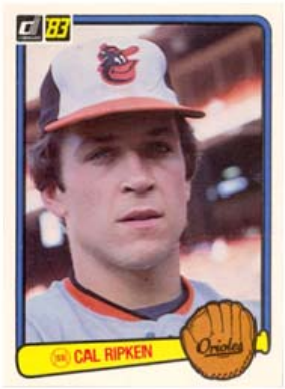
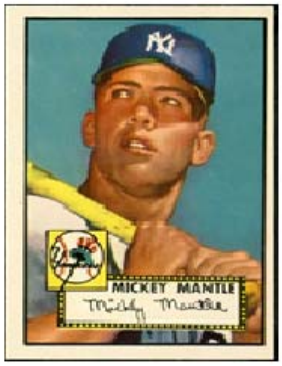
NCAAPMT Calculus Challenge 2011-2012

Challenge #5 **Due: November 30, 2011**

Since the Thanksgiving Holidays are coming up, this project is due after that break.

When to Sell the Baseball Card?





A dealer in sports cards has a rare baseball card, and he's trying to decide when to sell it. He knows its value will grow over time, but he could sell the card and invest the money in a bank account, and the value of the money would also grow over time due to interest. The question is: when should the dealer sell the card?

Experience suggests to the dealer that over time, the value of the card, like many collectibles, will grow in a way consistent with the following model:

$$V(t) = Ce^{k\sqrt{t}},$$

where $C > 0$ and $k > 0$ are constants, V is the value of the baseball card in dollars, and t is the number of years after the present time.

1. Does $V(t)$ have a maximum value for some t? If so, interpret this value. If not, explain how you know a maximum does not exist.

2. Does $V(t)$ have a point of inflection for some t? If so, interpret this value. If not, explain how you know a point of inflection does not exist.

Suppose that the dealer, who is 18 years old, decides to sell the card at time t, sometime in the next 42 years: $0 \le t \le 42$. At that time t, he'll invest the money he gets for the sale of the card in a bank account that earns an interest rate of r, compounded continuously. When he turns 65, he'll take the money that's in his bank account and begin to draw on it for his retirement. Let A be the amount of money in his account when he turns 65.

3. Explain why the amount of money in the dealer's account when he turns 65 can be modeled by

$$A(t) = \left(Ce^{k\sqrt{t}}\right)e^{r\cdot(42-t)}.$$

4. Let $C = 2500$, $k = 0.5$, and $r = 0.06$. When should the dealer sell the card so as to maximize the amount in his retirement account when he turns 65?

5. Using $C = 2500$ and $r = 0.06$, plot the function $A(t)$ for several different values of *k*. What does a larger value of k imply about the value of the card over time? And now, what does a larger value of k imply about the best time to sell the card? Do these two facts seem consistent with one another?

6. Using $C = 2500$ and $k = 0.5$, plot the function $A(t)$ for several different values of r. What does a larger value of r imply about the best time to sell the card? Is that consistent with the meaning of r?

7. Let t_{opt} be the optimal time to sell the baseball card—*i.e.*, the time that will maximize $A(t)$. Use calculus to find t_{opt} in the general model (in terms of *C*, *r*, and *k*).

8. Are the properties of t_{opt} as it relates to k and r consistent with those you found in steps 5 and 6 above? Explain your answer.