

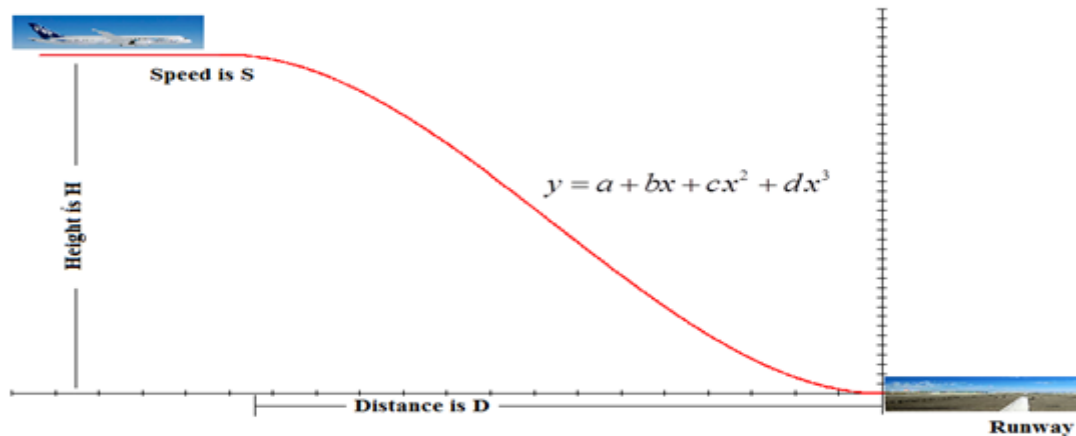
NCAAPMT Calculus Challenge 2011-2012

Challenge #6

Due: Friday, December 16, 2011

“Please return all seatbacks to their upright and locked positions. We will be landing parametrically.”

An airliner is making its final landing approach and follows a cubic path, $y = a + bx + cx^2 + dx^3$, to smoothly transition from its horizontal flight path at altitude H to a horizontal landing path at ground level. Several conditions must be met to insure that the descent and landing are comfortable for the passengers.



- The cruising altitude is H until the transition to the descent curve occurs at $(-D, H)$.
So, $\frac{dy}{dx} = 0$ at $(-D, H)$ and also at the landing site at $(0, 0)$.
- The horizontal airspeed, $\frac{dx}{dt}$, is a constant S mph throughout the descent (this is somewhat unrealistic, but is OK as an approximation).
- At no time during the descent should the vertical component of acceleration exceed, in absolute value, some fixed constant k , $0 \leq k \ll g$, where $g = 32 \text{ ft/sec}^2$ is the acceleration constant for gravity.
 - Find values of a , b , c , and d in the function $y = a + bx + cx^2 + dx^3$ which satisfy the conditions above in terms of D and S .
 - Determine the maximum vertical acceleration under the stated conditions in terms of the parameters in the problem (D, H, S, k) .
 - Suppose $S = 600$ mph, $H = 35,000$ feet, and $k = 0.4 \text{ ft/sec}^2$. From what distance must the plane begin its descent?
 - Suppose a small single engine plane is landing on a runway in the West Virginia mountains. It must fly over a mountain and immediately descend to the runway in the valley below. In this case, we have $S = 150$ mph, $H = 12,000$ feet, and $D = 10$ miles. What is the value of k for this landing?