NCAAPMT Calculus Challenge 2011-2012

Challenge #7 Due: January 25, 2012

Wrapped Roller Coaster

The artist Christo was famous for very large-scale, temporary, abstract art installations, such as wrapping large buildings and monuments entirely in fabric. Let's imagine a Christo-style art project we'll call "Wrapped Roller Coaster". We would like to wrap the space *beneath* a roller coaster entirely in fabric—that is, we want sheets of fabric to extend from the roller coaster straight down to the ground below, at all points on the coaster. *(The wrapped Reichstag in Berlin, 1995)*

We find an amusement park willing to cooperate with us, letting us rent out one of their roller coasters for one week. We pick a roller coaster that is not too complicated: when the sun is directly overhead, the shadow of the roller coaster forms a simple closed curve, never intersecting itself.

Using a digital camera and a panoramic photo app, you digitally splice together different photos of the roller coaster so that you get a final photo showing how it would look stretched out from end to end. Your photo is shown below in Figure 1. For convenience, markers are added to the photo at 50-meter increments, and the vertical scale is stretched relative to the horizontal scale.

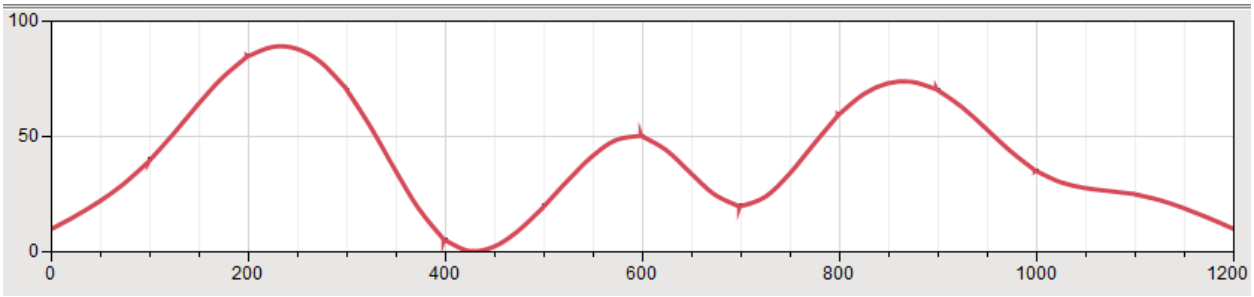


Figure 1

1) Use the photo to approximate the area of fabric you'll need. Be sure to explain your method and state whether you think your approximation is too large or too small.

Exhibition Hall

After the success of your art installation "Wrapped Roller Coaster," a museum asks you to help design a new exhibition hall for them. Here's the floor plan you are considering (all units in meters).

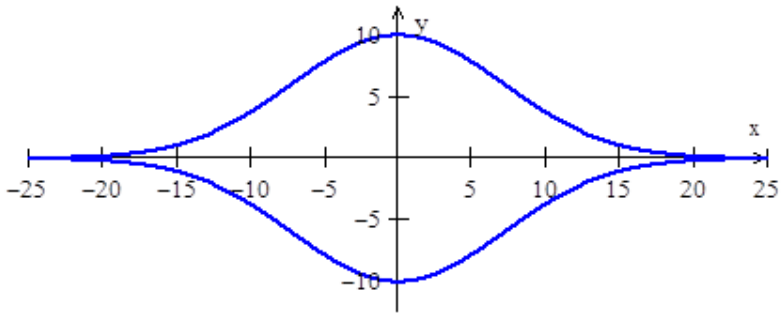
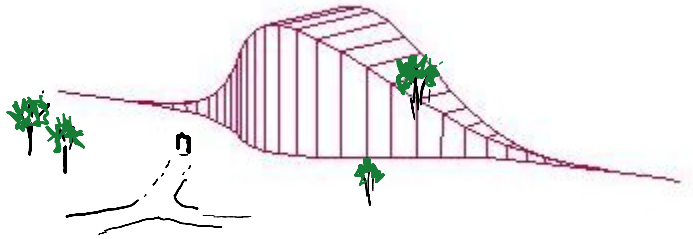


Figure 2: $f(x)=10e^{-\frac{x^2}{100}}$ and $g(x)=-10e^{-\frac{x^2}{100}}$

The boundaries of the building are modeled by the functions *f* and *g* shown above on the domain $[-25, 25]$. You would like to build an exhibition hall whose base is the region defined above and whose vertical cross-sections are all perfect squares. A computer-generated rendering of the structure is shown below.



Since it's critical that an art museum maintain a constant interior temperature, the managers are concerned about the interior volume of the exhibition hall.

2) Compute the total interior volume of the proposed exhibition hall.

3) Determine the average height of the building.

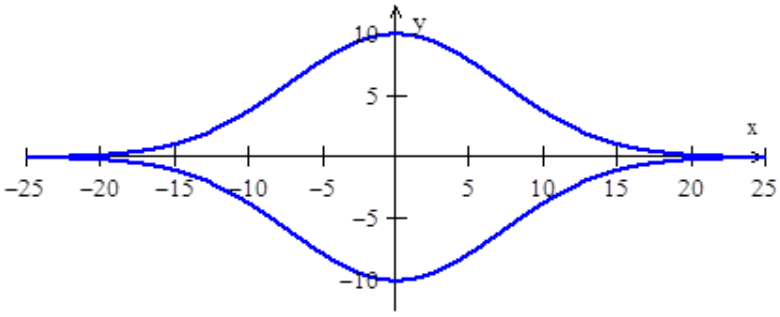
Building the exhibition hall

Your plan for the exhibition hall is a go! You determine that it will take 24 months to build it. However, the number of workers required is not constant over that time period. You estimate that if $t = 0$ is the time when construction begins, then the number of workers required at month t (where $0 \le t \le 24$) is approximately given by the function $w(t) = -0.07t^3 + 3t^2 - 45t + 500$. It costs \$3000 to hire a worker for a month. (We say that \$3000 is the cost of a "worker-month".)

4) Estimate how much it will cost to employ all the workers that it will take to build the exhibition hall.

Grand Opening!

The exhibition hall is about to open! For the grand opening, you've decided to string lanterns completely around the perimeter of the building. Since vertical cross-sections of the building are squares, the footprint of the building on the ground has a similar profile to the vertical profile; it's as wide as it is tall at every point:



$$f(x) = 10e^{-\frac{x^2}{100}} \text{ and } g(x) = -10e^{-\frac{x^2}{100}}$$

Lanterns will be placed at every 50 cm (0.5 meters) around the entire perimeter of the building.

5) Determine how many lanterns will be needed.

Thanks to Floyd Bullard, Instructor of Mathematics, NC School of Science and Mathematics, for this bi-week's problem.